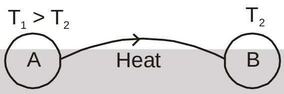
# CALORIMETRY AND THERMAL EXPANSION

## 1. HEAT

The energy that is being transferred between two bodies or between adjacent parts of a body as a result of temperature difference is called heat. Thus, heat is a form of energy. It is energy in transit whenever temperature differences exist. Once it is transferred, it becomes the internal energy of the receiving body. It should be clearly understood that the word "heat" is meaningful only as long as the energy is being transferred. Thus, expressions like "heat in a body" or "heat of a body" are meaningless.




When we say that a body is heated it means that its molecules begin to move with greater kinetic energy.

S.I. unit of heat energy is joule (J). Another common unit of heat energy is calorie (cal).

**1 calorie = 4.18 joules.**

**1 calorie :** The amount of heat needed to increase the temperature of 1 gm of water from 14.5 to 15.5 ºC at one atmospheric pressure is 1 calorie.

### 1.1 Mechanical Equivalent of Heat

In early days heat was not recognized as a form of energy. Heat was supposed to be something needed to raise the temperature of a body or to change its phase. Calorie was defined as the unit of heat. A number of experiments were performed to show that the temperature may also be increased by doing mechanical work on the system. These experiments established that heat is equivalent to mechanical energy and measured how much mechanical energy is equivalent to a calorie. If mechanical work W produces the same temperature change as heat H, we write, **W = JH**

where J is called mechanical equivalent of heat. J is expressed in joule/calorie. The value of J gives how many joules of mechanical work is needed to raise the temperature of 1 g of water by 1°C.

## *Solved Examples*

**Example 1.** What is the change in potential energy (in calories) of a 10 kg mass after 10 m fall ?

**Solution :** Change in potential energy

$\Delta U = mgh = 10 \times 10 \times 10 = 1000 \text{ J} = \frac{1000}{4.186} \text{ cal}$ **Ans.**

## 2. SPECIFIC HEAT

Specific heat of substance is equal to heat gain or released by that substance to raise or fall its temperature by 1ºC for a unit mass of substance.

When a body is heated, it gains heat. On the other hand, heat is lost when the body is cooled. The gain or loss of heat is directly proportional to:

(a) the mass of the body $\Delta Q \propto m$

(b) rise or fall of temperature of the body $\Delta Q \propto \Delta T$

$$\Delta Q \propto m\,\Delta T \quad \text{or} \quad \Delta Q = m\,s\,\Delta T \quad \text{or} \quad dQ = m\,s\,dT \quad \text{or} \quad Q = \int m\,s\,dT.$$

where s is a constant and is known as the specific heat of the body $s = \frac{Q}{m\Delta T}$. S.I. unit of s is joule/kg-Kelvin and C.G.S. unit is cal./gm °C.

**Specific heat of water :** S = 4200 J/kgºC = 1000 cal/kgºC = 1 Kcal/kgºC = 1 cal/gmºC

**Specific heat of steam = half of specific heat of water = specific heat of ice**

## Solved Examples

**Example 2.** Heat required to increases the temperate of 1 kg water by 20ºC

**Solution :** heat required = $\Delta Q = ms\Delta\theta$

$\because$ S = 1 cal/gmºC = 1 Kcal/kgºC = 1 × 20 = 20 Kcal.

### 2.1 Heat capacity or Thermal capacity :

Heat capacity of a body is defined as the amount of heat required to raise the temperature of that body by 1°. If 'm' is the mass and 's' the specific heat of the body, then **Heat capacity = ms**.

Units of heat capacity in: CGS system is, **cal ºC$^{-1}$**; SI unit is, **JK$^{-1}$**

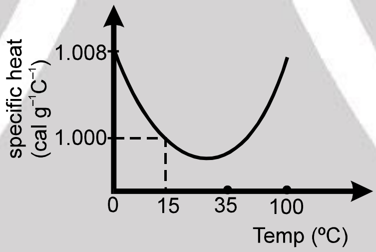
### 2.2 Important Points:

**(a)** We know, $s = \frac{Q}{m\Delta T}$, if the substance undergoes the change of state which occurs at constant temperature ($\Delta T = 0$), then s = Q/0 = $\infty$. Thus the specific heat of a substance when it melts or boils at constant temperature is infinite.

**(b)** If the temperature of the substance changes without the transfer of heat (Q = 0) then $s = \frac{Q}{m\Delta T} = 0$. Thus when liquid in the thermos flask is shaken, its temperature increases without the transfer of heat and hence the specific heat of liquid in the thermos flask is zero.

**(c)** To raise the temperature of saturated water vapours, heat (Q) is withdrawn. Hence, specific heat of saturated water vapours is negative. (This is for your information only and not in the course)

**(d)** The slight variation of specific heat of water with temperature is shown in the graph at 1 atmosphere pressure. Its variation is less than 1% over the interval form 0 to 100ºC.




### 2.3 Relation between Specific heat and Water equivalent :

It is the amount of water which requires the same amount of heat for the same temperature rise as that of the object

$ms\,\Delta T = m_W\,S_W\,\Delta T \Rightarrow m_W = \frac{ms}{s_W}$

In calorie $s_W = 1$ $\therefore$ $m_W = ms$

$m_w$ is also represent by W

So W = ms.

### 2.4 Phase change :

Heat required for the change of phase or state,

**Q = mL**, L = latent heat.

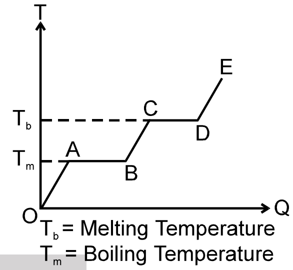
**Latent heat (L):** The heat supplied to a substance which changes its state at constant temperature is called latent heat of the body.

**Latent heat of Fusion ($L_f$) :** The heat supplied to a substance which changes it from solid to liquid state at its melting point and 1 atm. pressure is called latent heat of fusion. Latent heat of fusion of ice is 80 kcal/kg

**Latent heat of vaporization ($L_v$):** The heat supplied to a substance which changes it from liquid to vapour state at its boiling point and 1 atm. pressure is called latent heat of vaporization. Latent heat of vaporization of water is 540 kcal $kg^{-1}$.

**Latent heat of ice :** L = 80 cal/gm = 80 Kcal/kg = 4200 × 80 J/kg

**Latent heat of steam :** L = 540 cal/gm = 540 Kcal/kg = 4200 × 540 J/kg




The given figure, represents the change of state by different lines

OA – solid state, AB – solid + liquid state (Phase change)

BC – liquid state, CD – liquid + vapour state (Phase change)

DE – vapour state

$\Delta Q = ms\Delta T$

slope $\frac{\Delta T}{\Delta Q} = \frac{1}{ms} \Rightarrow \frac{\Delta T}{\Delta Q} \propto \frac{1}{S}$

where mass (m) of substance constant slope of T – Q graph is inversely proportional to specific heat, if in given diagram

(slope) OA > (slope) DE

then $(s)_{OA} < (s)_{DE}$

when $\Delta Q = mL$

If (length of AB) > (length of CD)

then (latent heat of AB) > (latent heat of CD)

## Solved Examples

**Example 3.** Find the amount of heat released if 1 kg steam at 200ºC is converted into –20ºC ice.

**Solution :** Heat released ΔQ = heat release to convert steam at 200 ºC into 100ºC steam + heat release to convert 100º C steam into 100ºC water + heat release to convert 100º water into 0ºC water + heat release to convert 0 ºC water into – 20ºC ice.

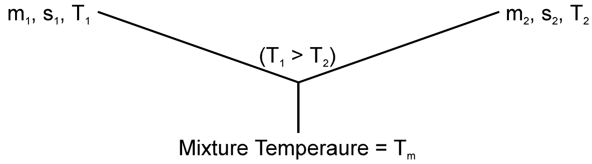
$$\Delta Q = 1 \times \frac{1}{2} \times 100 + 540 \times 1 + 1 \times 1 \times 100 + 1 \times 80 + 1 \times \frac{1}{2} \times 20 = 780 \text{ Kcal.}$$

## 3. CALORIMETRY

The branch of thermodynamics which deals with the measurement of heat is called calorimetry. A simple calorimeter is a vessel generally made of copper with a stirrer of the same material. The vessel is kept in a wooden box to isolate it thermally from the surrounding. A thermometer is used to measure the temperature of the contents of the calorimeter. Object at different temperatures are made to come in contact with each other in the calorimeter. As a result, heat is exchanged between the object as well as with the calorimeter. Neglecting any heat exchange with the surrounding.

### 3.1 Law of Mixture:

When two substances at different temperatures are mixed together, then exchange of heat continues to take place till their temperatures become equal. This temperature is then called final temperature of mixture. Here, **Heat taken by one substance = Heat given by another substance**




$\Rightarrow m_1 s_1 (T_1 - T_m) = m_2 s_2 (T_m - T_2)$

## Solved Example

**Example 4.** An iron block of mass 2 kg, fall from a height 10 m. After colliding with the ground it loses 25% energy to surroundings. Then find the temperature rise of the block. (Take sp. heat of iron 470 J/kg ºC)

**Solution :** $mS\Delta\theta = \frac{1}{4}mgh \quad \Rightarrow \quad \Delta\theta = \frac{10 \times 10}{4 \times 470}$

### Zeroth law of thermodynamics :

If objects A and B are separately in thermal equilibrium with a third object C, then objects A and B are in thermal equilibrium with each other.

## Solved Example

**Example 5.** The temperature of equal masses of three different liquids A, B, and C are 10ºC 15ºC and 20ºC respectively. The temperature when A and B are mixed is 13ºC and when B and C are mixed, it is 16ºC. What will be the temperature when A and C are mixed?

**Solution :** When A and B are mixed $mS_1 \times (13 - 10) = m \times S_2 \times (15 - 13)$

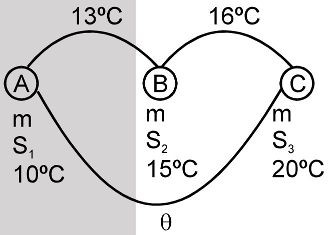
$3S_1 = 2S_2$ .....(1)

when B and C are mixed

$S_2 \times 1 = S_3 \times 4$ .....(2)

when C and A are mixed

$S_1(\theta - 10) = S_3 \times (20 - \theta)$ .....(3)




by using equation (1), (2) and (3)

we get $\theta = \frac{140}{11}$ ºC

**Example 6.** If three different liquid of different masses specific heats and temperature are mixed with each other and then what is the temperature mixture at thermal equilibrium.

$m_1, s_1, T_1 \rightarrow$ specification for liquid

$m_2, s_2, T_2 \rightarrow$ specification for liquid

$m_3, s_3, T_3 \rightarrow$ specification for liquid.

**Solution :** Total heat lost or gain by all substance is equal to zero

$\Delta Q = 0$

$m_1s_1(T - T_1) + m_2s_2(T - T_2) + m_3s_3 (T - T_3) = 0$

So $T = \frac{m_1s_1T_1 + m_2s_2T_2 + m_3s_3T_3}{m_1s_1 + m_2s_2 + m_3s_3}$

**Example 7.** In following equation calculate value of H 1 kg ice at –20ºC = H + 1 Kg water at 100ºC, here H means heat required to change the state of substance.

**Solution :** Heat required to convert 1 kg ice at – 20ºC into 1 kg water at 100ºC

= 1 kg ice at – 20ºC to 1 kg ice at 0ºC ice at 0ºC + 1 kg water

at 0ºC + 1 kg water at 0ºC to 1 kg water at 100ºC

$= 1 \times \frac{1}{2} \times 20 + 1 \times 80 + 1 \times 100 = 190$ Kcal. So H = – 190 Kcal

Negative sign indicate that 190 Kcal heat is with drawn from 1 kg water at 100ºC to convert it into 1 kg ice at – 20ºC

**Example 8.** 1kg ice at –20ºC is mixed with 1 kg steam at 200ºC. Then find equilibrium temperature and mixture content.

**Solution :** Let equilibrium temperature is 100 ºC heat required to convert 1 kg ice at –20ºC to 1 kg water at 100ºC is equal to

$H_1 = 1 \times \frac{1}{2} \times 20 + 1 \times 80 + 1 \times 1 \times 100 = 190$ Kcal

heat release by steam to convert 1 kg steam at 200ºC to 1 kg water at 100ºC is equal to

$H_2 = 1 \times \frac{1}{2} \times 100 + 1 \times 540 = 590$ Kcal

1 kg ice at – 20ºC = $H_1$ + 1kg water at 100ºC .....(1)

1 kg steam at 200ºC = $H_2$ + 1kg water at 100ºC .....(2)

by adding equation (1) and (2)

1 kg ice at –20ºC + 1 kg steam at 200ºC = $H_1 + H_2$ + 2 kg water at 100ºC.

Here heat required to ice is less than heat supplied by steam so mixture equilibrium temperature is 100ºC then steam is not completely converted into water.

So mixture has water and steam which is possible only at 100ºC mass of steam which converted into water is equal to

$$m = \frac{190 - 1 \times \frac{1}{2} \times 100}{540} = \frac{7}{27} \text{ kg}$$

so mixture content

mass of steam = $1 - \frac{7}{27} = \frac{20}{27}$ kg

mass of water = $1 + \frac{7}{27} = \frac{34}{27}$ kg

## 4. THERMAL EXPANSION

Most materials expand when their temperature is increased. Rails roads tracks, bridges all have some means of compensating for thermal expansion. When a homogeneous object expands, the distance between any two points on the object increases. Figure shows a block of metal with a hole in it. **The expanded object is like a photographic enlargement.** That in the hole expands in the same proportion as the metal, it does not get smaller

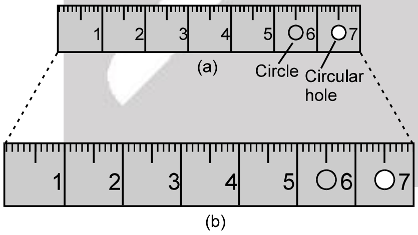
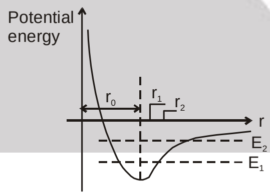



Fig. The same steel ruler two different temperatures. When it expands, the scale, the numbers, the thickness, and the diameters of the circle and circular hole are all incrased by the same factor. (The expansion has been exaggerated for clarity.)



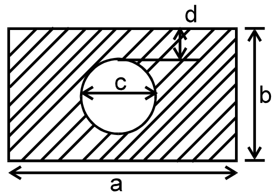

Thermal expansion arises because the well is not symmetrical about the equilibrium position $r_0$. As the temperature rise the energy of the atom increases. The average position when the energy is $E_2$ is not the same as that when the energy is $E_1$.

At the atomic level, thermal expansion may be understood by considering how the potential energy of the atoms varies with distance. The equilibrium position of an atom will be at the minimum of the potential energy well if the well is symmetric. At a given temperature each atom vibrates about its equilibrium position and its average remains at the minimum point. If the shape of the well is not symmetrical the average position of an atom will not be at the minimum point. When the temperature is raised the amplitude of the vibrations increases and the average position is located at a greater inter atomic separation. This increased separation is manifested as expansion of the material.

Almost all solids and liquids expand as their temperature increases. Gases also expand if allowed. Solids can change in length, area or volume, while liquids change in their volumes.

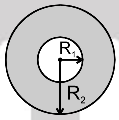
## Solved Example

**Example 9.** A rectangular plate has a circular cavity as shown in the figure. If we increase its temperature then which dimension will increase in following figure.




**Solution :** Distance between any two point on an object increases with increase in temperature. So, all dimension a, b, c and d will increase

**Example 10.** In the given figure, when temperature is increased then which of the following increases

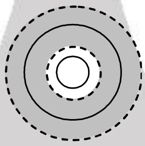



(A) $R_1$ (B) $R_2$ (C) $R_2 - R_1$

**Solution :** All of the above

- - - - - represents expanded Boundary

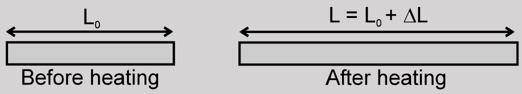
--------- represents original Boundary



As the intermolecular distance between atoms increases on heating hence the inner and outer perimeter increases. Also if the atomic arrangement in radial direction is observed then we can say that it also increases hence all A, B, C are true.

## 5. LINEAR EXPANSION

When the rod is heated, its increase in length $\Delta L$ is proportional to its original length $L_0$ and change in temperature $\Delta T$ where $\Delta T$ is in ºC or K.



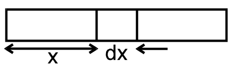

$dL = \alpha L_0\, dT \quad \Rightarrow \quad \Delta L = \alpha\, L_0\, \Delta T \quad$ If $\alpha\, \Delta T << 1$

$\alpha = \dfrac{\Delta L}{L_0 \Delta T}$ where $\alpha$ is called the coefficient of linear expansion whose unit is $ºC^{-1}$ or $K^{-1}$.

$L = L_0 (1 + \alpha\, \Delta\, T)$. Where L is the length after heating the rod.

### Variation of $\alpha$ with temperature and distance

**(a)** If $\alpha$ varies with distance, $\alpha = ax + b$.




Then total expansion = $\int (ax + b)\, \Delta T\, dx$.

**(b)** If $\alpha$ varies with temperature, $\alpha = f(T)$. Then $\Delta L = \int \alpha\, L_0\, dT$

**Note :** Actually thermal expansion is always 3-D expansion. When other two dimensions of object are negligible with respect to one, then observations are significant only in one dimension and it is known as linear expansion.

## Solved Example

**Example 11.** What is the percentage change in length of 1m iron rod if its temperature changes by 100ºC. $\alpha$ for iron is $2 \times 10^{-5}$/ºC.

**Solution :** percentage change in length due to temperature change $\%\ell = \times 100 = \alpha\Delta\theta \times 100$
$= 2 \times 10^{-5} \times 100 \times 100 = 0.2\%$ **Ans.**

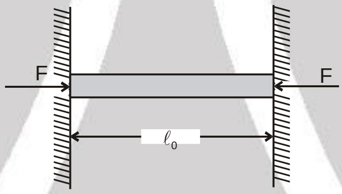
### 5.1 Thermal stress of a material :

If the rod is free to expand then there will be no stress and strain. Stress and strain is produced only when an object is restricted to expand or contract according to change in temperature. When the temperature of the rod is decreased or increased under constrained condition, compressive or tensile stresses are developed in the rod. These stresses are known as thermal stresses.

**Strain** $= \dfrac{\Delta L}{L_0} = \dfrac{\text{final length – original length}}{\text{original length}} = \alpha\,\Delta T$

**Note :** Original and final length should be at same temperature.

Consider a rod of length $\ell_0$ which is fixed between to rigid end separated at a distance $\ell_0$ now if the temperature of the rod is increased by $\Delta\theta$ then the strain produced in the rod will be :




$$\text{strain} = \frac{\text{length of the rod at new temperature-natural length of the rod at new temperature}}{\text{natural length of the rod at new temperature}}$$

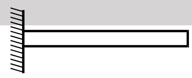
$$= \frac{\ell_0 - \ell_0(1+\alpha\Delta\theta)}{\ell_0(1+\alpha\Delta\theta)} = \frac{-\ell_0\alpha\Delta\theta}{\ell_0(1+\alpha\Delta\theta)}$$

$\because$ $\alpha$ is very small so

strain = $-\alpha\Delta\theta$ (negative sign in the answer represents that the length of the rod is less than the natural length that means is compressed by the ends.)

## Solved Example

**Example 12.** In the given figure a rod is free at one end and other end is fixed. When we change the temperature of rod by $\Delta\theta$, then strain produced in the rod will be



(A) $\alpha\Delta\theta$ (B) $\frac{1}{2}\alpha\Delta\theta$ (C) zero (D) information incomplete

**Solution :** Here rod is free to expand from one side by so by changing temperature no strain will be produced in the rod. Hence ans. is (C)

**Example 13.** An iron ring measuring 15.00 cm in diameter is to be shrunk on a pulley which is 15.05 cm in diameter. All measurements refer to the room temperature 20°C. To what minimum temperature should the ring be heated to make the job possible? Calculate the strain developed in the ring when it comes to the room temperature. Coefficient of linear expansion of iron = $12 \times 10^{-6}$/°C.

**Solution :** The ring should be heated to increase its diameter from 15.00 cm to 15.05 cm.

Using $\ell_2 = \ell_1 (1 + \alpha\,\Delta\theta)$,

$$= \frac{0.05\,\text{cm}}{15.00\,\text{cm} \times 12 \times 10^{-6}\,/^\circ\text{C}} = 278°\text{C}$$

The temperature = 20°C + 278°C = 298°C.

The strain developed = $\frac{\ell_2 - \ell_1}{\ell_1}$ = $3.33 \times 10^{-3}$.

**Example 14.** A steel rod of length 1m rests on a smooth horizontal base. If it is heated from 0°C to 100°C, what is the longitudinal strain developed?

**Solution :** in absence of external force no strain or stress will be created hear rod is free to move.

**Example 15.** A steel rod is clamped at its two ends and rests on a fixed horizontal base. The rod is in natural length at 20°C. Find the longitudinal strain developed in the rod if the temperature rises to 50°C. Coefficient of linear expansion of steel = $1.2 \times 10^{-5}$/°C.

**Solution :** as we known that strain

strain = $\frac{\text{change in length}}{\text{original length}} = \frac{\Delta\ell}{\ell_0}$ $\therefore$ Strain = $\alpha\Delta\theta = 1.2 \times 10^{-5} \times (50 - 20) = 3.6 \times 10^{-4}$

here strain is compressive strain because final length is smaller than initial length.

**Example 16.** A steel wire of cross-sectional area 0.5 mm$^2$ is held between two fixed supports. If the wire is just taut at 20°C, determine the tension when the temperature falls to 0°C. Coefficient of linear expansion of steel is $1.2 \times 10^{-5}$/°C and its Young's modulus is $2.0 \times 10^{11}$ N/m$^2$.

**Solution :** here final length is more than original length so that strain is tensile and tensile force is given by

$F = AY\,\alpha\,\Delta t = 0.5 \times 10^{-6} \times 2 \times 10^{11} \times 1.2 \times 10^{-5} \times 20$ **=** 24 N

## 5.2 Variation of time period of pendulum clocks :

The time represented by the clock hands of a pendulum clock depends on the number of oscillation performed by pendulum every time it reaches to its extreme position the second hand of the clock advances by one second that means second hand moves by two seconds when one oscillation in complete

Let $T = 2\pi\sqrt{\frac{L_0}{g}}$ at temperature $\theta_0$ and $T' = 2\pi\sqrt{\frac{L}{g}}$ at temperature $\theta$.

$$\frac{T'}{T} = \sqrt{\frac{L'}{L}} = \sqrt{\frac{L\left[1+\alpha\Delta\theta\right]}{L}} = 1 + \frac{1}{2}\alpha\,\Delta\,\theta$$

Therefore change (loss or gain) in time per unit time lapsed is $\frac{T'-T}{T} = \frac{1}{2}\,\alpha\,\Delta\theta$

gain or loss in time in duration of 't' in

**$\Delta t = \frac{1}{2}\,\alpha\,\Delta\theta\,t$,** if T is the correct time then

(a) $\theta < \theta_0$, $T' < T$ clock becomes fast and gain time

(b) $\theta > \theta_0$, $T' > T$ clock becomes slow and loose time

## Solved Examples

**Example 17.** A pendulum clock consists of an iron rod connected to a small, heavy bob. If it is designed to keep correct time at 20°C, how fast or slow will it go in 24 hours at 40°C? Coefficient of linear expansion of iron = $1.2 \times 10^{-6}$/°C.

**Solution :** The time difference occurred in 24 hours (86400 seconds) is given by

$\Delta t = \frac{1}{2}\ \alpha\ \Delta\theta\ t = \frac{1}{2} \times 1.2 \times 10^{-6} \times 20 \times 86400 = 1.04$ sec. **Ans.**

This is loss of time as $\theta$ is greater than $\theta_0$. As the temperature increases, the time period also increases. Thus, the clock goes slow.

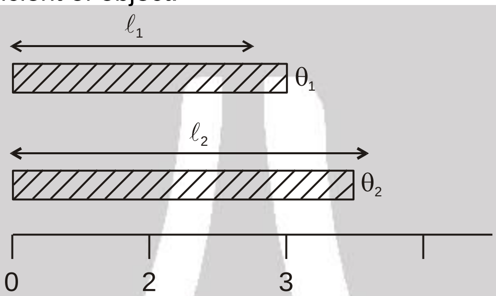
## 5.3 Measurement of length by metallic scale :

**Case (i) :** When object is expanded only $\ell_2 = \ell_1\ \{1 + \alpha_0(\theta_2 - \theta_1)$

$\ell_1$ = actual length of object at $\theta_1$ºC = measure length of object at $\theta_1$ºC.

$\ell_2$ = actual length of object at $\theta_2$ºC = measure length of object at $\theta_2$ºC.

$\alpha_0$ = linear expansion coefficient of object.



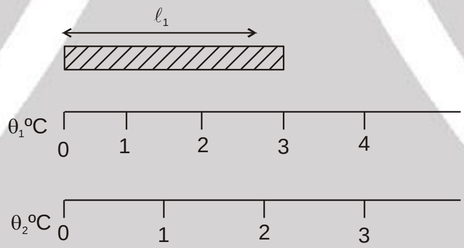

**Case (ii) :** When only measurement instrument is expanded actual length of object will not change but measured value (MV) decreases.

MV = $\ell_1\ \{1 - \alpha_S\ (\theta_2 - \theta_1)\}$

$\alpha_S$ = linear expansion coefficient of measuring instrument.

at $\theta_1$ °C MV = 3

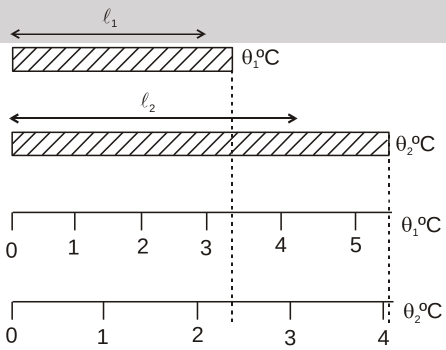
at $\theta_2$ °C MV = 2.2




**Case (iii) :** If both expanded simultaneously MV = $\{1 + (\alpha_0 - \alpha_s)\ (\theta_2 - \theta_1)$

(i) If $\alpha_0 > \alpha_s$, then measured value is more then the actual value at $\theta_1$ºC

(ii) If $\alpha_0 < \alpha_s$, then measured value is less then the actual value at $\theta_1$ºC




at $\theta_1$ºC MV = 3.4

$\theta_2$ºC MV = 4.1

**Measured value = calibrated value × {1 + $\alpha\ \Delta\ \theta$}**

where $\alpha = \alpha_0 - \alpha_s$

$\alpha_o$ = coefficient of linear expansion of object material, $\alpha_s$ = coefficient of linear expansion of scale material

$\Delta\theta = \theta - \theta_C$

$\theta$ = temperature at the time of measurement $\theta_C$ = temperature at the time of calibration.

For scale, **true measurement = scale reading [1 + $\alpha$ ($\theta - \theta_0$)]**

If **$\theta > \theta_0$ true measurement > scale reading**

**$\theta < \theta_0$ true measurement < scale reading**

## Solved Examples

**Example 18.** A bar measured with a Vernier caliper is found to be 180mm long. The temperature during the measurement is 10ºC. The measurement error will be if the scale of the Vernier caliper has been graduated at a temperature of 20ºC : ($\alpha = 1.1 \times 10^{-5}$ °C$^{-1}$. Assume that the length of the bar does not change.)

(A) $1.98 \times 10^{-1}$ mm (B) $1.98 \times 10^{-2}$ mm (C) $1.98 \times 10^{-3}$ mm (D) $1.98 \times 10^{-4}$ mm

**Answer :** **(B)**

**Solution :** True measurement = scale reading $[1 + \alpha(\theta - \theta_0)] = 180\ [1 - 10 \times 1.1 \times 10^{-5}]$

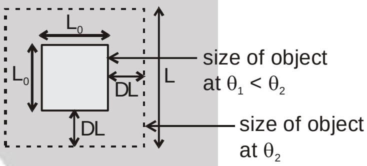
error = $180 - 180\ [1 - 1.1 \times 10^{-4}] = 1.98 \times 10^{-2}$ mm

## 6. SUPERFICIAL OR AREAL EXPANSION

When a solid is heated and its area increases, then the thermal expansion is called superficial or areal expansion. Consider a solid plate of area $A_0$. When it is heated, the change in area of the plate is directly proportional to the original area $A_0$ and the change in temperature $\Delta T$.

$dA = \beta A_0\, dT$ or **$\Delta A = \beta\, A_0\, \Delta T$**

$\beta = \dfrac{\Delta A}{A_0 \Delta T}$ Unit of $\beta$ is ºC$^{-1}$ or K$^{-1}$.




$A = A_0(1 + \beta\Delta T)$

where A is area of the plate after heating,

## Solved Example

**Example 19.** A plane lamina has area 2m$^2$ at 10ºC then what is its area at 110ºC It's superficial expansion is $2 \times 10^{-5}$/C

**Solution :** $A = A_0(1 + \beta\Delta\theta) = 2\{1 + 2 \times 10^{5} \times (110 - 10)\}$

$= 2 \times \{1 + 2 \times 10^{-3}\}$ **Ans.**

## 7. VOLUME OR CUBICAL EXPANSION

When a solid is heated and its volume increases, then the expansion is called volume expansion or cubical expansion. Let us consider a solid or liquid whose original volume is $V_0$. When it is heated to a new volume, then the change $\Delta V$

$dV = \gamma V_0\, dT$ or **$\Delta V = \gamma\, V_0\, \Delta T$**

$\gamma$ = Unit of $\gamma$ is ºC$^{-1}$ or K$^{-1}$.

**$V = V_0(1 + \gamma\Delta T)$**

where V is the volume of the body after heating

## Solved Example

**Example 20.** The volume of glass vessel is 1000 cc at 20°C. What volume of mercury should be poured into it at this temperature so that the volume of the remaining space does not change with temperature? Coefficient of cubical expansion of mercury and glass are $1.8 \times 10^{-4}$/°C and $9.0 \times 10^{-6}$/°C respectively.

**Solution :** Let volume of glass vessel at 20ºC is $V_g$ and volume of mercury at 20ºC is $V_m$
so volume of remaining space is = $V_g - V_m$
It is given constant so that

$V_g - V_m = V_g' - V'_m$

where $V_o'$ and $V_m'$ are final volumes.

$V_g - V_m = V_g \{1 + \gamma_g \Delta\theta\} - V_m\{1 + \gamma_{Hg} \Delta\theta\} \Rightarrow V_g \gamma_g = V_m \gamma_{Hg}$

$\Rightarrow V_m = \frac{100 \times 9 \times 10^{-6}}{1.8 \times 10^{-4}} \Rightarrow V_m = 50$ cc.

## 8. RELATION BETWEEN α , β AND γ

(i) For isotropic solids: **$\alpha : \beta : \gamma = 1 : 2 : 3$ or $\frac{\alpha}{1} = \frac{\beta}{2} = \frac{\gamma}{3}$**

(ii) For non-isotropic solid $\beta = \alpha_1 + \alpha_2$ and $\gamma = \alpha_1 + \alpha_2 + \alpha_3$. Here $\alpha_1$, $\alpha_2$ and $\alpha_3$ are coefficient of linear expansion in X, Y and Z direction.

## Solved Example

**Example 21.** If percentage change in length is 1% with change in temperature of a cuboid object ($\ell \times 2\ell \times 3\ell$) then what is percentage change in its area and volume.

**Solution :** percentage change in length with change in temperature = % $\ell$

$\frac{\Delta\ell}{\ell} \times 100 = \alpha\Delta\theta \times 100 = 1$

change in area

$\Rightarrow \% A = \frac{\Delta A}{A} \times 100 = \beta\Delta\theta \times 100 \Rightarrow 2(\alpha\Delta\theta \times 100)$

% A = 2 % **Ans.**

change in volume

$\% V = \frac{\Delta V}{V} \times 100 = V \Delta \theta \times 100 = 3 (\alpha \Delta \theta \times 100)$

% V = 3 % **Ans.**

## 9. VARIATION OF DENSITY WITH TEMPERATURE

As we known that mass = volume × density.

Mass of substance does not change with change in temperature so with increase of temperature, volume increases so density decreases and vice-versa.

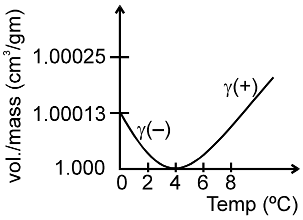
$d = \frac{d_0}{(1+\gamma\Delta T)}$.

For solids values of $\gamma$ are generally small so we can write $d = d_0 (1 - \gamma \Delta T)$ (using binomial expansion).

**Note :** (i) $\gamma$ for liquids are in order of $10^{-3}$.

(ii) **Anamolous expansion of water :**

For water density increases from 0 ºC to 4 ºC so $\gamma$ is negative and for 4 ºC to higher temperature $\gamma$ is positive. At 4 ºC density is maximum. This anamolous behaviour of water is due to presence of three types of molecules i.e. $H_2O$, $(H_2O)_2$ and $(H_2O)_3$ having different volume/mass at different temperatures.




This anomalous behaviour of water causes ice to form first at the surface of a lake in cold weather. As winter approaches, the water temperature decreases initially at the surface. The water there sinks because of its increase density. Consequently, the surface reaches $0^0$C first and the lake becomes covered with ice. Aquatic life is able to survive the cold winter as the lake bottom remains unfrozen at a temperature of about $4^0$C.

## Solved Example

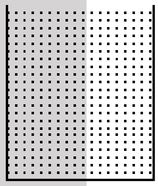
**Example 22.** The densities of wood and benzene at 0°C are 880 kg/m$^3$ and 900 kg/m$^3$ respectively. The coefficients of volume expansion are 1.2 × $10^{-3}$/°C for wood and 1.5 × $10^{-3}$/°C for benzene. At what temperature will a piece of wood just sink in benzene?

**Solution :** At just sink gravitation force = up thrust force

$\Rightarrow$ $mg = F_B$ $\Rightarrow$ $V\rho_1 g = V\rho_2 g$ $\Rightarrow$ $\rho_1 = \rho_2$

$$\Rightarrow \frac{880}{1+1.2\times10^{-3}\theta} = \frac{900}{1+1.5\times10^{-3}\theta} \quad \Rightarrow \quad \theta = 83^o\,C$$

## 10. APPARENT EXPANSION OF A LIQUID IN A CONTAINER



Initially container was full. When temperature change by $\Delta T$,

volume of liquid $V_L = V_0\,(1 + \gamma_L\,\Delta T)$

volume of container $V_C = V_0\,(1 + \gamma_C\,\Delta T)$

So overflow volume of liquid relative to container $\Delta V = V_L - V_C$

$\mathbf{\Delta V = V_0\,(\gamma_L - \gamma_C)\,\Delta T}$

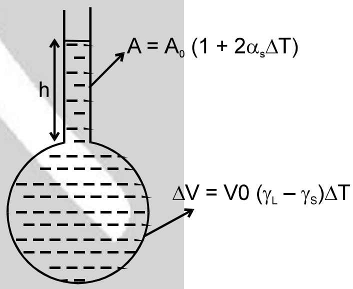
So, coefficient of apparent expansion of liquid w.r.t. container

$\gamma_{apparent} = \gamma_L - \gamma_C$.

In case of expansion of liquid + container system:

if $\gamma_L > \gamma_C \longrightarrow$ level of liquid rise

if $\gamma_L < \gamma_C \longrightarrow$ level of liquid fall




Increase in height of liquid level in tube when bulb was initially not completely filled

$$h = \frac{\text{volume of liquid}}{\text{area of tube}} = \frac{V_0(1+\gamma_L\Delta T)}{A_0(1+2\alpha_S\Delta T)} = h_0\,\{1 + (\gamma_L - 2\alpha_S)\,\Delta T\}$$

$\mathbf{h = h_0\,\{1 + (\gamma_L - 2\alpha_S)\,\Delta T\}}$

where $h_0$ = original height of liquid in container

$\alpha_S$ = linear coefficient of expansion of container.

## Solved Example

**Example 23.** A glass vessel of volume 100 cm$^3$ is filled with mercury and is heated from 25°C to 75°C. What volume of mercury will overflow? Coefficient of linear expansion of glass = 1.8 × $10^{-6}$/°C and coefficient of volume expansion of mercury is 1.8 × $10^{-4}$/°C.

**Solution :** $\Delta V = V_0(\gamma_L - \gamma_C)\,\Delta T = 100 \times \{1.8 \times 10^{-4} - 3 \times 1.8 \times 10^{-6}\} \times 50$

$\Delta V = 0.87$ cm$^3$ Ans.

## 11. VARIATION OF FORCE OF BUOYANCY WITH TEMPERATURE

If body is submerged completely inside the liquid
For solid, Buoyancy force

$F_B = V_0 d_L g$

$V_0$ = Volume of the solid inside liquid,

$d_L$ = density of liquid

Volume of body after increase its temperature $V = V_0 [1 + \gamma_S \Delta\theta]$,

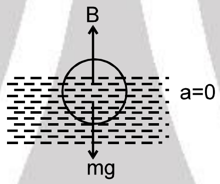
Density of body after increase its temperature $d'_L = \dfrac{d_L}{[1+\gamma_L\Delta\theta]}$.

Buoyancy force of body after increase its temperature, $F'_B = V d'_L g$, $\dfrac{F'_B}{F_B} = \dfrac{[1+\gamma_S\Delta\theta]}{[1+\gamma_L\Delta\theta]}$,

if $\gamma_S < \gamma_L$ then $F'_B < F_B$
(Buoyant force decreases) or apparent weight of body in liquid gets increased
$[W - F'_B > W - F_B]$ .

## Solved Example

**Example 24.** A body is float inside liquid if we increases temperature then what changes occur in Buoyancy force. (Assume body is always in floating condition)




**Solution :** Body is in equilibrium
So mg = B
and gravitational force does not change with change in temperature. So Buoyancy force remains constant.
By increasing temperature density of liquid decreases so volume of body inside the liquid increases to kept the Buoyance force constant for equal to gravitational force)

**Example 25.** In previous question discuss the case when body move downward, upwards and remains at same position when we increases temperature.

**Solution :** Let f = fraction of volume of body submerged in liquid.

$$f = \frac{\text{volume of body submerged in liquid}}{\text{total volume of body}}$$

$f_1 = \dfrac{v_1}{v_0}$ at $\theta_1$ºC

$f_2 = \dfrac{v_2}{v_0(1+3\alpha_S\Delta\theta)}$ at $\theta_2$ºC

for equilibrium $mg = B = v_1 d_1 g = v_2 d_2 g$.

so $v_2 = \dfrac{v_1 d_1}{d_2}$ $\because$ $d_2 = \dfrac{d_1}{1+\gamma_L\Delta\theta}$ $= v_1(1 + \gamma_L \Delta\theta)$ $\therefore$ $f_2 = \dfrac{v_1(1+\gamma_L\Delta\theta)}{v_0(1+3\alpha_s\Delta\theta)}$

where $\Delta\theta = \theta_2 - \theta_1$

**Case I :** Body move downward if $f_2 > f_1$
means $\gamma_L > 3\alpha_S$

**Case II :** Body move upwards if $f_2 < f_1$
means $\gamma_L < 3\alpha_S$

**Case III :** Body remains at same position
if $f_2 = f_1$
means $\gamma_L = 3\alpha_S$

## 12. BIMETALLIC STRIP

It two strip of different metals are welded together to form a bimetallic strip, when heated uniformly it bends in form of an arc, the metal with greater coefficient of linear expansion lies on convex side. The radius of arc thus formed by bimetal is :

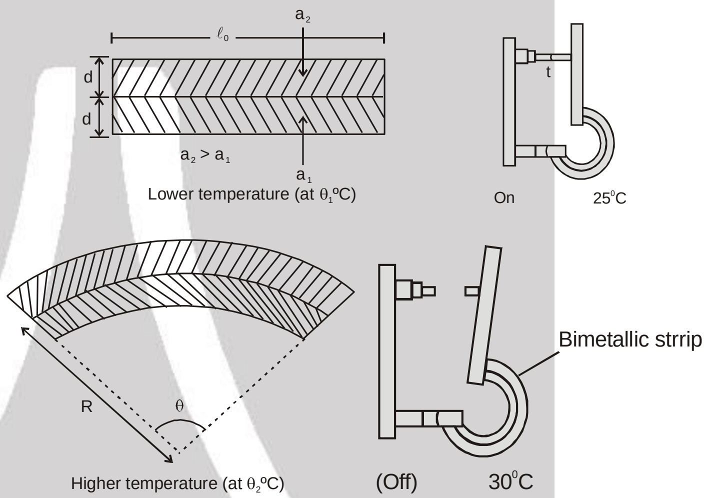
$$\ell_0 (1 + \alpha_1 \Delta\theta) = \left(R - \frac{d}{2}\right)\theta$$

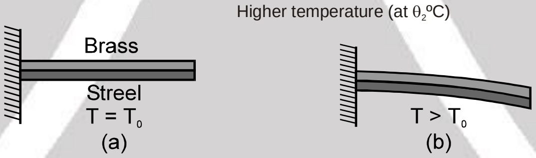
$$\ell_0 (1 + \alpha_2 \Delta\theta) = \left(R + \frac{d}{2}\right)\theta$$

$$\Rightarrow \quad \frac{1+\alpha_2\Delta\theta}{1+\alpha_1\Delta\theta} = \frac{R+\frac{d}{2}}{R-\frac{d}{2}}$$

$$\Rightarrow \quad R = \frac{d}{(\alpha_2 - \alpha_1)\Delta\theta}$$

$\Delta\theta$ = change in temperature
$= \theta_2 - \theta_1$




A bimetallic strip, consisting of a strip of brass and a strip of steel welded together, at temperature $T_0$ in figure (a) and figure (b). The strip bends as shown at temperatures above the reference temperature. Below the reference temperature the strip bends the other way. Many thermostats operate on this principle, making and breaking an electrical circuit as the temperature rises and falls.

## 13. APPLICATIONS OF THERMAL EXPANSION

(a) A small gap is left between two iron rails of the railway.
(b) Iron rings are slipped on the wooden wheels by heating the iron rings
(c) Stopper of a glass bottle jammed in its neck can be taken out by heating the neck.
(d) The pendulum of a clock is made of invar [an alloy of zinc and copper].

## 14. TEMPERATURE

Temperature may be defined as the **degree of hotness or coldness** of a body. Heat energy flows from a body at higher temperature to that at lower temperature until their temperatures become equal. At this stage, the bodies are said to be in thermal equilibrium.

### 14.1 Measurement of Temperature

The branch of thermodynamics which deals with the measurement of temperature is called thermometry. A thermometer is a device used to measure the temperature of a body. The substances like liquids and gases which are used in the thermometer are called thermometric substances.

## 14.2 Different Scales of Temperature

A thermometer can be graduated into following scales.

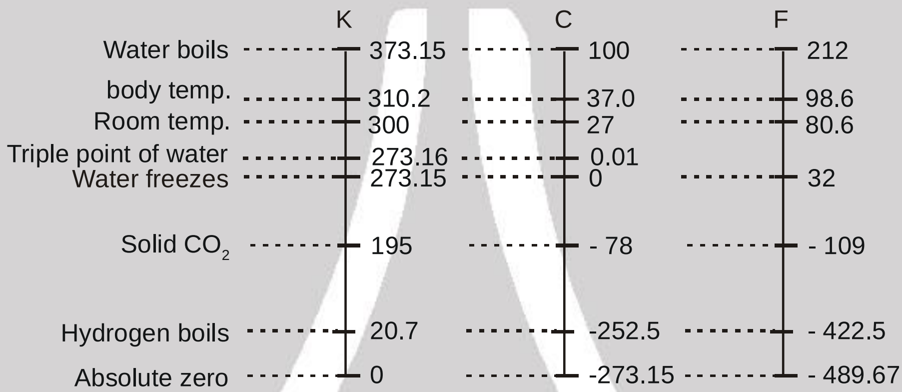
(a) The Centigrade or Celsius scale (ºC)

(b) The Fahrenheit scale (ºF)

(c) The Reaumer scale (ºR)

(d) Kelvin scale of temperature (K)

## 14.3 Comparison between Different Temperature Scales




The formula for the conversion between different temperature scales is:

$$\frac{K-273}{100} = \frac{C}{100} = \frac{F-32}{180} = \frac{R}{80}$$

General formula for the conversion of temperature from one scale to another:

$$\frac{\text{Temp on one scale}(S_1)\text{-Lower fixed point }(S_1)}{\text{Upper fixed point }(S_2) - \text{Lower fixed point }(S_1)}$$

$$= \frac{\text{Temp. on other scale}(S_2)\text{-Lower fixed point }(S_2)}{\text{Upper fixed point }(S_2) - \text{Lower fixed point }(S_2)}$$

## 14.4 Thermometers

Thermometers are device that are used to measure temperatures. All thermometers are based on the principle that some physical property of a system changes as the system temperature changes.

Required properties of good thermometric substance.

(1) Non-sticky (absence of adhesive force)

(2) Low melting point (in comparison with room temperature)

(3) High boiling temperature

(4) Coefficient of volumetric expansion should be high (to increase accuracy in measurement).

(5) Heat capacity should be low.

(6) Conductivity should be high

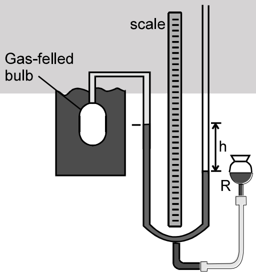
Mercury (Hg) suitably exhibits above properties.

## 14.5 Types of Thermometers

| Type of thermometer and its range | Thermometric property | Advantages | Disadvantages | Particular Uses |
|---|---|---|---|---|
| Mercury-in-glass – 39ºC to 450ºC | Length of column of mercury in capillary tube | (i) Quick and easy to (direct reading)<br>(ii) Easily portable | (i) Fragile<br>(ii) Small size limits<br>(iii) Limited range | (i) Every laboratory use where high accuracy is not required.<br>(ii) Can be calibrated against constant-volume gas thermometer for more accurate work |
| Constant-volume gas thermometer –270º to 1500ºC | Pressure of a fixed mass of gas at constant volume | (i) Very accurate<br>(ii) Very sensitive<br>(iii) Wide range<br>(iv) Easily reproducible | (i) Very large volume of bulb<br>(ii) Slow to use and inconvenient | (i) Standard against which others calibrated<br>(ii) He, $H_2$ or $N_2$ used depending on range<br>(iii) can be corrected to the ideal gas scale<br>(iv) Used as standard below-183ºC |
| Platinum resistance –180º to 1150ºC | Electrical resistance of a platinum coil | (i) Accurate<br>(ii) Wide range | Not suitable for varying temperature (i.e., is slow to respond to changes) | (i) Best thermometer for small steady temperature differences<br>(ii) Used as standard between 183ºC and 630ºC. |
| Thermocouple –250ºC to 1150ºC | Emf produced between junctions of dissimilar metals at different temperatures for measurement of emfs | (i) Fast response because of low heat capacity.<br>(ii) wide range<br>(iii) can be employed for remote readings using long leads. | Accuracy is lost if emf is measured using a moving-coil voltmeter (as may be necessary for rapid changes when potentiometer is unsuitable) | (i) Best thermometer for small steady temperature differences<br>(ii) Can be made direct reading by calibrating galvanometer<br>(iii) Used as standard between 630ºC and 1063ºC |
| Radiation pyrometer above 1000ºC | Colour of radiation emitted by a hot body | Does not come into contact when temperature is measured | (i) Cumbersome<br>(ii) Not direct reading (needs a trained observer) | (i) Only thermometer possible for very high temperatures<br>(ii) Used as standard above 1063ºC. |

## 14.6 The constant-volume gas thermometer

The standard thermometer, against which all other thermometers are calibrated, is based on the pressure of a gas in a fixed volume. Figure shows such a constant volume gas thermometer; it consists of a gas-filled bulb connected by a tube to a mercury monometer.




A constant volume gas thermometer, its bulb immersed in a liquid whose temperature T is to be measured.

$$T = (273.16\ K)\left(\lim_{gas \to 0} \frac{P}{P_3}\right)$$

P = Pressure at the temperature being measured, $P_3$ = pressure when bulb in a triple point cell.

## Solved Example

**Example 26.** The readings of a thermometer at 0ºC and 100ºC are 50 cm and 75 cm of mercury column respectively. Find the temperature at which its reading is 80 cm of mercury column?

**Solution :** By using formula $\frac{80-50}{75-50} = \frac{T-0}{100-0}$ $\Rightarrow$ T = 120ºC

## Solved Miscellaneous Problems

**Problem 1.** A bullet of mass 10 gm in moving with speed 400m/s. Find its kinetic energy in calories ?

**Solution :** $\Delta k = \frac{1}{2} \times \frac{10}{1000} \times 400 \times 400 = 800$ $\Rightarrow$ $\frac{800}{4.2} = 191.11$ Cal.

**Problem 2.** Calculate amount of heat required to convert 1 kg steam from 100ºC to 200ºC steam

**Solution :** Heat required = $1 \times \frac{1}{2} \times 100 = 50$ kcal

**Problem 3.** Calculate heat required to raise the temperature of 1 g of water through 1ºC ?

**Solution :** heat required = $1 \times 10^{-3} \times 1 \times 1 = 1 \times 10^{-3}$ kcal = 1 cal

**Problem 4.** 420 J of energy supplied to 10 g of water will raise its temperature by

**Solution :** $\frac{420 \times 10^{-3}}{4.20} = 10 \times 10^{-3} \times 1 \times \Delta t = 10^{\circ}$ C

**Problem 5.** The ratio of the densities of the two bodies is 3 : 4 and the ratio of specific heats is 4 : 3. Find the ratio of their thermal capacities for unit volume ?

**Solution :** $\frac{\rho_1}{\rho_2} = \frac{3}{4}$, $\frac{s_1}{s_2} = \frac{4}{3}$

ratio = $\frac{m \times s}{m/\rho}$ $\Rightarrow$ $\frac{\theta_1}{\theta_2} = \frac{s_1}{s_2} \times \frac{\rho_1}{\rho_2} = 1 : 1$.

**Problem 6.** Heat releases by 1 kg steam at 150ºC if it convert into 1 kg water at 50ºC.

**Solution :** $H = 1 \times \frac{1}{2} \times 50 + 1 \times 540 + 1 \times 1 \times 50 = 540 + 75 = 615$ Kcal

Heat release = 615 Kcal.

**Problem 7.** 200 gm water is filled in a calorimetry of negligible heat capacity. It is heated till its temperature is increase by 20ºC. Find the heat supplied to the water.

**Solution :** $H = 200 \times 10^{-3} \times 1 \times 20 = 4$ Kcal.

Heat supplied = 4000 cal

**Problem 8.** A bullet of mass 5 gm is moving with speed 400 m/s. strike a target and energy. Then calculate rise of temperature of bullet. Assuming all the lose in kinetic energy is converted into heat energy of bullet if its specific heat is. 500J/kgºC.

**Solution :** Kinetic energy = $\frac{1}{2} \times 5 \times 10^{-3} \times 400 \times 400$

$ms\,\Delta T = 5 \times 10^{-3} \times 500 \times \Delta T$

$\Delta T = 160^{\circ}$ C

Rise in temperature is 160 ºC

**Problem 9.** 1 kg ice at –10ºC is mixed with 1 kg water at 100ºC. Then find equilibrium temperature and mixture content.

**Solution :** Heat taken by 1 kg Ice = Heat given by 1 kg water

$1 \times \frac{1}{2} \times 10 + 1 \times 80 + 1 \times T = 1 \times (100 - T)$

$85 = 100 - 2T \Rightarrow 2T = 15$

$\theta = \frac{15}{2} = 7.5\ ^{o}C$, water

**Problem 10.** 1kg ice at –10º is mixed with 1kg water at 50ºC. Then find equilibrium temperature and mixture content.

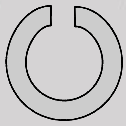
**Solution :** Heat taken by ice = 5 Kcal + 80 Kcal = 85 Kcal ; Heat given by water = 1 × 1 × 50 = 50 Kcal
Heat taken > Heat given so, ice will not complete melt let m g ice melt then

$1 \times \frac{1}{2} \times 10 + 80\,m = 50$

$80\,m = 45 \Rightarrow m = \frac{45}{80}$

Content of mixture $\left\{\begin{matrix} \text{water} & \left(1+\frac{45}{80}\right) & \text{kg} \\ \text{ice} & \left(1-\frac{45}{80}\right) & \text{kg} \end{matrix}\right\}$ and temperature is 0ºC

**Problem 11.** A small ring having small gap is shown in figure on heating what will happen to size of gap.



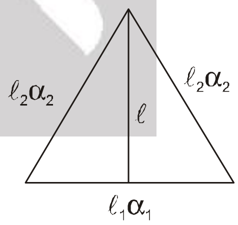
**Solution :** Gap will also increase. The reason is same as in above example.

**Problem 12.** An isosceles triangle is formed with a thin rod of length $\ell_1$ and coefficient of linear expansion $\alpha_1$, as the base and two thin rods each of length $\ell_2$ and coefficient of linear expansion $\alpha_2$ as the two sides. If the distance between the apex and the midpoint of the base remain unchanged as the temperature is varied show that $\frac{\ell_1}{\ell_2} = 2\sqrt{\frac{\alpha_2}{\alpha_1}}$ .

**Solution :**

$\ell = \sqrt{\left(\frac{\ell_1}{2}\right)^2 + (\ell_2)^2}$

$\ell^2 = \left(\frac{\ell_1}{2}\right)^2 + (\ell_2)^2$




$0 = \frac{2\ell_1}{2}\frac{1}{2}\frac{d\ell_1}{dT} + 2\ell_2 \times \frac{d\ell_2}{dT} = 2\ell_2 \times \ell_2\ \alpha_2\Delta T$

$\Rightarrow \frac{\ell_1^2}{\ell_2^2} = 4\ \frac{\alpha_2}{\alpha_1} \Rightarrow \frac{\ell_1}{\ell_2} = 2\sqrt{\frac{\alpha_2}{\alpha_1}}$ .

**Problem 13.** A concrete slab has a length of 10 m on a winter night when the temperature is 0°C. Find the length of the slab on a summer day when the temperature is 35°C. The coefficient of linear expansion of concrete is $1.0 \times 10^{-5}$ /°C.

**Solution :** $\ell_t = 10(1 + 1 \times 10^{-5} \times 35) = 10.0035$ m

**Problem 14.** A steel rod is clamped at its two ends and rests on a fixed horizontal base. The rod is unstrained at 20°C. Find the longitudinal strain developed in the rod if the temperature rises to 50°C. Coefficient of linear expansion of steel = $1.2 \times 10^{-5}$/°C.

**Solution :** $\frac{\Delta\ell}{\ell} = \frac{\ell_0\alpha\Delta\theta}{\ell_0} = 3.6 \times 10^{-4}$

**Problem 15.** If rod is initially compressed by $\Delta\ell$ length then what is the strain on the rod when the temperature
(a) is increased by $\Delta\theta$ (b) is decreased by $\Delta\theta$.

**Solution:** (a) Strain = $\frac{\Delta\ell}{\ell} + \alpha\Delta\theta$ (b) Strain = $\left|\frac{\Delta\ell}{\ell} - \alpha\Delta\theta\right|$

**Problem 16.** A pendulum clock having copper rod keeps correct time at 20°C. It gains 15 seconds per day if cooled to 0°C. Calculate the coefficient of linear expansion of copper.

**Solution :** $\frac{15}{24\times60\times60} = \frac{1}{2}\alpha \times 20 \Rightarrow \alpha = \frac{1}{16\times3600} = 1.7 \times 10^{-5}/°C$

**Problem 17.** A meter scale made of steel is calibrated at 20°C to give correct reading. Find the distance between 50 cm mark and 51 cm mark if the scale is used at 10°C. Coefficient of linear expansion of steel is $1.1 \times 10^{-5}/°C$.

**Solution :** $\ell_t = 1\,(1 - 1.1 \times 10^{-5} \times 10) = 0.99989$ cm

**Problem 18.** A uniform solid brass sphere is rotating with angular speed $\omega_0$ about a diameter. If its temperature is now increased by 100ºC. What will be its new angular speed. (Given $\alpha_B = 2.0 \times 10^{-5}$ per-ºC)

(A) $\frac{\omega_0}{1-0.002}$ (B) $\frac{\omega_0}{1+0.002}$ (C*) $\frac{\omega_0}{1+0.004}$ (D) $\frac{\omega_0}{1-0.004}$

**Solution :** $I_0\,\omega_0 = I_t\,\omega t$

$Mr_0^2\,\omega_0 = Mr_0^2\,(1 + 2\alpha\Delta T)\omega_t$ ; $\omega_t = \frac{\omega_0}{1+0.004}$.

**Problem 19.** The volume occupied by a thin - wall brass vessel and the volume of a solid brass sphere are the same and equal to 1,000 cm$^3$ at 0ºC. How much will the volume of the vessel and that of the sphere change upon heating to 20ºC? The coefficient of linear expansion of brass is $\alpha = 1.9 \times 10^{-5}$.

**Solution :** $\Delta V = V_0\, 3\alpha\, \Delta T = 1.14$ cm$^3$
1.14 cm$^3$ for both

**Problem 20.** A thin copper wire of length L increases in length by 1%, when heated from temperature $T_1$ to $T_2$. What is the percentage change in area when a thin copper plate having dimensions 2L × L is heated from $T_1$ to $T_2$ ?
(A) 1% (B) 3% (C) 4% (D*) 2%

**Solution :** $L_f = L\,(1 + \alpha\Delta t) \Rightarrow \frac{L_f}{L} \times 100 = (1 + \alpha\Delta t) \times 100 = 1\%$

$A_f = 2L \times L\,(1 + 2\,\alpha\Delta t) \Rightarrow \frac{A_f}{2L\times L} \times 100 = (1 + 2\alpha\,\Delta t) \times 100 = 2\%$

**Problem 21.** The density of water at 0°C is 0.998 g/cm$^3$ and at 4°C is 1.000 g/cm$^3$. Calculate the average coefficient of volume expansion of water in the temperature range 0 to 4°C.

**Solution :** $d_t = \frac{d_0}{1+\gamma\Delta t} \Rightarrow 1 = \frac{0.998}{1+\gamma\times4} \Rightarrow \gamma = -5 \times 10^{-4}\,/\,^0C$

**Problem 22.** A glass vessel measures exactly 10 cm × 10 cm × 10 cm at 0°C. it is filled completely with mercury at this temperature. When the temperature is raised to 10°C, 1.6 cm$^3$ of mercury overflows. Calculate the coefficient of volume expansion of mercury. Coefficient of linear expansion of glass = $6.5 \times 10^{-6}/°C$

**Solution :** $\Delta V = V_{Hg} - V_V$

$1.6 = 10^3 \gamma_\ell \times 10 - 10^3 \times 3 \times 6.5 \times 10^{-6} \times 10$

$\gamma_L = (1.6 + 0.195) \times 10^{-4} = 1.795 \times 10^{-4}\ /^0C$

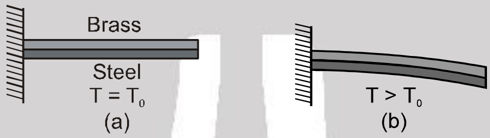
**Problem 23.** A metal ball immersed in alcohol weighs $W_1$ at $0^0C$ and $W_2$ at $50^0C$. The coefficient of cubical expansion of the metal is less than alcohol. Assuming that density of the metal is large compared to that of the alcohol, find which of $W_1$ and $W_2$ is greater?

**Solution :** $\gamma_M < \gamma_\ell$ so, $\frac{F'_B}{F_B} = \frac{[1+\gamma_S \Delta\theta]}{[1+\gamma_\ell \Delta\theta]} \Rightarrow F'_B < F_B$

so Apparent weight increases

so, $W_2 > W_1$

**Problem 24.** In figure which strip brass or steel have higher coefficient of linear expansion.




**Solution :** Brass Strip

**Problem 25.** The upper and lower fixed points of a faulty thermometer are 5ºC and 105ºC. If the thermometer reads 25ºC, what is the actual temperature ?

**Solution :** $\frac{25-5}{100} = \frac{C-0}{100}$

C = 20º C

**Problem 26.** At what temperature is the Fahrenheit scale reading equal to twice of Celsius ?

**Solution :** $\frac{F-32}{180} = \frac{C-0}{100}$

$\frac{2x-32}{180} = \frac{x-0}{100}$

10x – 160 = 9x

x = 160º C

**Problem 27.** Temperature of a patient is 40º C. Find the temperature on Fahrenheit scale ?

**Solution :** $\frac{F-32}{180} = \frac{40-0}{100} \Rightarrow F = 104^o F$

# Exercise-1

✍ **Marked Questions can be used as Revision Questions.**

## PART - I : SUBJECTIVE QUESTIONS

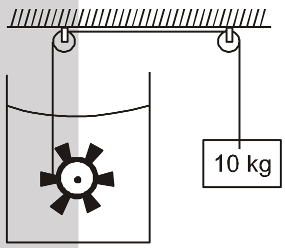
### Section (A) : Calorimetry

**A-1.**✍ In the following equation calculate the value of H.
1 kg steam at 200ºC = H + 1 kg water at 100ºC ($S_{steam}$ = Constant = 0.5 Cal/gm°C)

**A-2.** From what height should a piece of ice (0°C) fall so that it melts completely? Only one-quarter of the energy produced is absorbed by the ice as heat. (Latent heat of ice = $3.4 \times 10^5$ J $kg^{-1}$, g = 10 $m/s^2$)

**A-3.** A copper cube of mass 200g slides down on a rough inclined plane of inclination 37º at a constant speed. Assume that any loss in mechanical energy goes into the copper block as thermal energy. Find the increase in the temperature of the block as it slides down 60 cm. Specific heat capacity of copper = 420 J/kg-K.

**A-4**✍**.** A paddle wheel is connected with a block of mass 10 kg as shown in figure. The wheel is completely immersed in liquid of heat capacity 4000 J/K. The container is adiabatic. For the time interval in which block goes down 1 m slowly calculate




(a) Work done on the liquid
(b) Heat supplied to the liquid
(c) Rise in the temperature of the liquid
Neglect the heat capacity of the container and the paddle. (g =10 $m/s^2$)

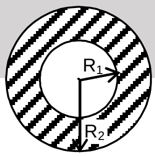
**A-5** 300 g of water at 25ºC is added to 100 g of ice at 0ºC. Find the final temperature of the mixture. **[1989; 2M]**

### Section (B) : Thermal Expansion

**B-1.** The temperature of a metal ball is raised. Arrange the percentage change in volume, surface area and radius in ascending order.

**B-2.** A brass disc fits in a hole in a steel plate. Would you heat or cool the system to loosen the disc from the hole? Assume that $\alpha_s < \alpha_b$.

**B-3.**✍ Temperature of plate is increased by $\Delta\theta$ then find new




(a) inner radius
(b) outer radius
(c) the difference in outer and inner radius and show that it is positive
(d) area of plate material (assume coefficient of expansion is $\alpha$ )

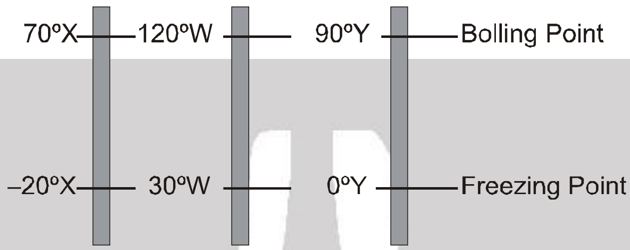
**B-4.**✍ We have a hollow sphere and a solid sphere of equal radii and of the same material. They are heated to raise their temperature by equal amounts. How will the change in their volumes, due to volume expansions, be related? Consider two cases (i) hollow sphere is filled with air, (ii) there is vacuum inside the hollow sphere.

**B-5** What should be the sum of lengths of an aluminium and steel rod at 0ºC is, so that at all temperatures their difference in length is 0.25m. (Take coefficient of linear expansion for aluminium and steel at 0°C as $22 \times 10^{-6}$ /ºC and $11 \times 10^{-6}$ /ºC respectively.)

**B-6** A steel tape is correctly calibrated at 20 ºC and is used to measure the length of a table at 30°C. Find the percentage error in the measurement of length. [$\alpha_{steel} = 11 \times 10^{-6}$/ ºC]

### Section (C) : Temperature

**C-1.** The figure shows three temperature scales with the freezing and boiling points of water indicated.




(a) Rank the size of a degree on these scales, greatest first.

(b) Rank the following temperatures, highest first : 50ºX, 50ºW and 50ºY.

**C-2 .** What is the temperature at which we get the same reading on both the centigrade and Fahrenheit scales?

## PART - II : SINGLE CHOICE OBJECTIVE QUESTIONS

### Section (A) : Calorimetry

**A-1.** A small quantity, mass m, of water at a temperature $\theta$ (inºC) is poured on to a large mass M of ice which is at its melting point. If c is the specific heat capacity of water and L the latent heat of fusion of ice, then the mass of ice melted is given by :

(A) $\frac{ML}{mc\theta}$ (B) $\frac{mc\theta}{ML}$ (C) $\frac{Mc\theta}{L}$ (D) $\frac{mc\theta}{L}$

**A-2.** A thermally isolated vessel contains 100 g of water at 0ºC. When air above the water is pumped out, some of the water freezes and some evaporates at 0ºC itself. Then the mass of the ice formed if no water is left in the vessel. Latent heat of vaporization of water at 0ºC = $2.10 \times 10^{6}$ J/kg and latent heat of fusion of ice = $3.36 \times 10^{5}$ J/kg.

(A) 86.2 g (B) 13.8 g (C) 76.2 g (D) 65.6 g

**A-3.** 20 gm ice at –10 ºC is mixed with m gm steam at 100 ºC. Minimum value of m so that finally all ice and steam converts into water. (Use $s_{ice}$ = 0.5 cal/gmºC, $s_{water}$ =1 cal/gmºC, L(melting)=80 cal/gm and L (vaporization) = 540 cal/gm)

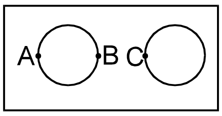
(A) $\frac{85}{32}$ gm (B) $\frac{85}{64}$ gm (C) $\frac{32}{85}$ gm (D) $\frac{64}{85}$ gm

**A-4.** 2 kg ice at – 20 ºC is mixed with 5 kg water at 20 ºC. Then final amount of water in the mixture will be : [Specific heat of ice = 0.5 cal/gm ºC, Specific heat of water = 1 cal/gm ºC, Latent heat of fusion of ice = 80 cal/gm ] **[JEE-2003 (Scr.), 3/84,–1]**

(A) 6 kg (B) 7 kg (C) 3.5 kg (D) 5 kg

## Section (B) : Thermal Expansion

**B-1.** Two large holes are cut in a metal sheet. If this is heated, distances AB and BC, (as shown)




(A) both will increase (B) both will decrease

(C) AB increases, BC decreases (D) AB decreases, BC increases

**B-2.** A steel scale is to be prepared such that the millimeter intervals are to be accurate within $6 \times 10^{-5}$ mm. The maximum temperature variation from the temperature of calibration during the reading of the millimeter marks is ($\alpha = 12 \times 10^{-6}$ /ºC)

(A) 4.0°C (B) 4.5°C (C) 5.0°C (D) 5.5°C

**B-3.** Expansion during heating

(A) occurs only in a solid

(B) increases the density of the material

(C) decreases the density of the material

(D) occurs at the same rate for all liquids and solids.

**B-4.** If a bimetallic strip is heated, it will

(A) bend towards the metal with lower thermal expansion coefficient.

(B) bend towards the metal with higher thermal expansion coefficient.

(C) twist itself into helix.

(D) have no bending

**B-5.** Two rods, one of aluminium and the other made of steel, having initial length $\ell_1$ and $\ell_2$ are connected together to form a single rod of length $\ell_1 + \ell_2$. The coefficients of linear expansion for aluminium and steel are $\alpha_a$ and $\alpha_s$ respectively. If the length of each rod increases by the same amount when their temperature are raised by tºC, then find the ratio $\frac{\ell_1}{(\ell_1 + \ell_2)}$. **[JEE-2003 (Scr.), 3/84,–1]**

(A) $\frac{\alpha_s}{\alpha_a}$ (B) $\frac{\alpha_a}{\alpha_s}$ (C) $\frac{\alpha_s}{(\alpha_a + \alpha_s)}$ (D) $\frac{\alpha_a}{(\alpha_a + \alpha_s)}$

**B-6.** A liquid with coefficient of volume expansion $\gamma$ is filled in a container of a material having the coefficient of linear expansion $\alpha$ .If the liquid overflows on heating, then –

(A) $\gamma > 3\alpha$ (B) $\gamma < 3\alpha$ (C) $\gamma = 3\alpha$ (D) none of these

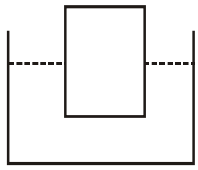
## Section (C) : Temperature

**C-1.** A difference of temperature of 25º C is equivalent to a difference of :

(A) 45º F (B) 72º F (C) 32º F (D) 25º F

## PART - III : MATCH THE COLUMN

**1.✍** A cylindrical isotropic solid of coefficient of linear expansion $\alpha$ and density $\rho$ floats in a liquid of coefficient of volume expansion $\gamma$ and density d as shown in the diagram



| **Column I** | **Column II** |
|---|---|
| (A) volume of cylinder inside the liquid remains constant | (p) $\gamma = 0$ |
| (B) volume of cylinder outside the liquid remains constant | (q) $\gamma = 2\alpha$ |
| (C) Height of cylinder outside the liquid remains constant | (r) $\gamma = 3\alpha \frac{d}{\rho}$ |
| (D) Height of cylinder inside the liquid remain constant | (s) $\gamma = (2\alpha + \alpha \frac{d}{\rho})$ |

**2.✍** In the following question column-I represents some physical quantities & column-II represents their units, match them

| **Column I** | **Column II** |
|---|---|
| (A) Coefficient of linear expansion | (p) Cal/°C |
| (B) Water equivalent | (q) gm |
| (C) heat capacity | (r) $(°C)^{-1}$ |
| (D) Specific heat | (s) Cal/g°C |

# Exercise-2

✍ **Marked Questions can be used as Revision Questions.**

## PART - I : ONLY ONE OPTION CORRECT TYPE

**1.✍** A metal ball of specific gravity 4.5 and specific heat 0.1 cal/gm-°C is placed on a large slab of ice at 0°C. When ball's temperature become 0°C then half of the ball sinks in the ice. The initial temperature of the ball is : (Latent heat capacity of ice = 80 cal/g, specific gravity of ice = 0.9)

(A) 100 °C (B) 90 °C (C) 80 °C (D) 70 °C

**2.** In a steel factory it is found that to maintain M kg of iron in the molten state at its melting point an input power P watt is required. When the power source is turned off, the sample completely solidifies in time t second. The latent heat of fusion of iron is

(A) 2 Pt / M (B) Pt / 2M (C) Pt / M (D) PM / t

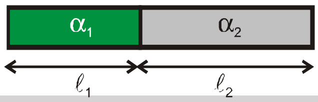
**3.** Steam at 100ºC is passed into 1.1 kg of water contained in a calorimeter of water equivalent 0.02 kg at 15ºC till the temperature of the calorimeter and its contents rises to 80ºC. The mass of the steam condensed in kilogram is : **[JEE 1986, 2]**

(A) 0.130 (B) 0.065 (C) 0.260 (D) 0.135

**4.** If I is the moment of inertia of a solid body having $\alpha$ -coefficient of linear expansion then the change in I corresponding to a small change in temperature $\Delta T$ is

(A) $\alpha I \Delta T$ (B) $\frac{1}{2}\alpha I \Delta T$ (C) $2\alpha I \Delta T$ (D) $3\alpha I \Delta T$

**5.** Two rods having length $\ell_1$ and $\ell_2$, made of materials with the linear coefficient of expansion $\alpha_1$ and $\alpha_2$, were welded together. The equivalent coefficients of linear expansion for the obtained rod :




(A) $\frac{\ell_1\alpha_2 + \ell_2\alpha_1}{\ell_1 + \ell_2}$ (B) $\frac{\ell_1\alpha_1 + \ell_2\alpha_2}{\alpha_1 + \alpha_2}$ (C) $\frac{\ell_1\alpha_1 + \ell_2\alpha_2}{\ell_1 + \ell_2}$ (D) $\frac{\ell_2\alpha_1 + \ell_1\alpha_2}{\alpha_1 + \alpha_2}$

**6.** The volume thermal expansion coefficient of an ideal gas at constant pressure is

(A) T (B) $T^2$ (C) $\frac{1}{T}$ (D) $\frac{1}{T^2}$

(Here T = absolute temperature of gas)

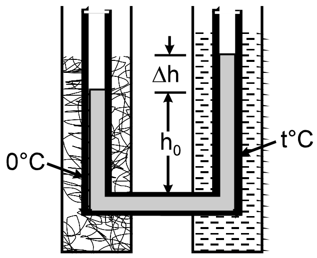
**7.** A metal ball immersed in water weighs $w_1$ at 5ºC and $w_2$ at 50ºC. The coefficient of cubical expansion of metal is less than that of water. Then

(A) $w_1 > w_2$ (B) $w_1 < w_2$ (C) $w_1 = w_2$ (D) data is insufficient

**8.** A piece of metal floats on mercury. The coefficient of volume expansion of the metal and mercury are $\gamma_1$ & $\gamma_2$ respectively. If the temperatures of both mercury and the metal are increased by an amount $\Delta T$, the fraction of the volume of the metal submerged in mercury changes by the factor of (Ratio of final fraction to the initial fraction) **[JEE 1991, 2]**

(A) $\frac{1+\gamma_2\Delta T}{1+\gamma_1\Delta T}$ (B) $\frac{1+\gamma_1\Delta T}{1+\gamma_2\Delta T}$ (C) $1 + (\gamma_1 + \gamma_2)\Delta T$ (D) None of these

**9.** Two vertical glass tubes filled with a liquid are connected at their lower ends by a horizontal capillary tube. One tube is surrounded by a bath containing ice and water at 0°C and the other by hot water at t°C. The difference in the height of the liquid in the two columns is $\Delta h$, and the height of the column at 0°C is $h_0$. Coefficient of volume expansion of the liquid is.




(A) $\frac{\Delta h}{h_0 t}$ (B) $\frac{2\Delta h}{h_0 t}$ (C) $\frac{2h_0}{\Delta h t}$ (D) $\frac{h_0}{\Delta h t}$

**10._** A small pond of depth 0.5 m deep is exposed to a cold winter with outside temperature of 263 K. Thermal conductivity of ice is $K = 2.2\ W\ m^{-1}\ K^{-1}$, latent heat $L = 3.4 \times 10^5\ Jkg^{-1}$ and density $\rho = 0.9 \times 10^3\ kgm^{-3}$. Take the temperature of the pond to be 273 K. The time taken for the whole pond to freeze is about. **[Olympiad (Stage-1) 2017]**
(A) 20 days (B) 25 days (C) 30 days (D) 35 days

**11._** Two rods identical in geometry but of different materials having co-efficient of thermal expansion $\alpha_1$ and $\alpha_2$ and Young's modulii $Y_1$ and $Y_2$ respectively are fixed between two rigid massive walls. The rods are heated such that they undergo the same increase in temperature. There is no bending of the rods. If $\alpha_1 : \alpha_2 = 2 : 6$ the thermal stresses developed in the two rods are equal provided $Y_1 : Y_2$ is equal to :
(A) 2 : 3 (B) 1 : 1 (C) 3 : 1 (D) 4 : 9

**12._** Two identical thin metal strips, one of aluminum and the other of iron are riveted together to form a bimetallic strip. The temperature is raised by 50º C. If the central planes of the two strips are separated by 2mm and the coefficients of thermal expansion of aluminum and iron are respectively $30 \times 10^{-6}/^0C$ and $10 \times 10^{-6}/^0C$ the average radius of curvature of the bimetallic strip is about. **[Olympiad 2014 (stage-1)]**
(A) 50 cm (B) 100 cm (C) 150 cm (D) 200 cm

**13._** Two thin rods of length $l_1$ and $l_2$ at a certain temperature are joined to each other end to end. The composite rod is then heated through a temperature $\theta$. The coefficients of linear expansion of the two rods are $\alpha_1$ and $\alpha_2$ respectively. Then, the effective coefficient of linear expansion of the composite rod is: **[Olympiad 2015 (stage-1)]**
(A) $\frac{\alpha_1 + \alpha_2}{2}$ (B) $\sqrt{\alpha_1 . \alpha_2}$ (C) $\frac{l_1\alpha_2 + l_2\alpha_1}{l_1 + l_2}$ (D) $\frac{l_1\alpha_1 + l_2\alpha_2}{l_1 + l_2}$

## PART - II : SINGLE AND DOUBLE VALUE INTEGER TYPE

**1.** A pitcher contains 20 kg of water. 0.5 gm of water comes out on the surface of the pitcher every second through the pores and gets evaporated taking energy from the remaining water. Calculate the approximate time (in min) in which temperature of the water decreases by 5ºC. Neglect backward heat transfer from the atmosphere to the water. (Write the answer to the nearest integer)
Specific heat capacity of water = 4200J/KgºC
Latent heat of vaporization of water $2.27 \times 10^6$ J/Kg

**2.** How long does a 59 kw water heater take to raise the temperature of 150 L of water from 21°C to 38ºC (in min)

**3.** The specifc heat of a substance varies with temperature according to $c = 0.2 + 0.16\ T + 0.024\ T^2$ with T in °c and c is cal/gk. Find the energy (in cal) requied to raise the temp of 2g substance from 0° to 5°C.

**4.** 50g of Ice at 0°C is mixed with 200g of water at 0°C. 6 kcal heat is given to system [Ice + water]. Find the temperature (in °C) of the system.

**5.** Earth receives 1400 W/m² of solar power. If all the solar energy falling on a lens of area 0.2 m² is focused on to a block of ice of mass 280 grams, the time taken to melt the ice will be X × 10 sec. Find the value of x. (Latent heat of fusion of ice = $3.3 \times 10^5$ J/kg) **[JEE 1997, 2]**

**6.** A 50 gm lead bullet, specific heat 0.02 cal/gm is initially at 30ºC. It is fired vertically upwards with a speed of 840 m/sec & on returning to the starting level strikes a cake of ice at 0º C. How much ice is melted. Assume that all energy is spent in melting only. [Latent heat of ice = 80 cal/gm]. Write the answer (in gms) to nearest integer. **[REE 1988, 5]**

**7.** The temperature of 100 gm of water is to be raised from $24^{o}$ C to $90^{o}$ C by adding steam to it. Calculate the mass of the steam (in gms) required for this purpose. **[JEE 1996, 2]**

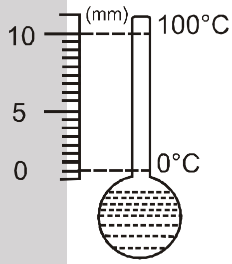
**8.** An electrical heating coil was placed in a calorimeter containing 360 gm of water at $10^{o}$ C. The coil consumes energy at the rate of 90 watt. The water equivalent of the calorimeter and the coil is 40 gm. Calculate what will be the temperature (in °C) of water after 10 minutes. Write the answer to nearest integer. J = 4.2 Joules/cal. **[REE 1985, 7]**

**9.**


As a result of temp rise of 32° C, a bar with a crack at its centre buckles upward. If the fixed distance $\ell_0$ is 4 m, and coefficient of linear expansion of bar in $25 \times 10^{-6}$ °c$^{-1}$.
Find the rise x (in cm) of the centre.

**10.** Level of a certain liquid at 0°C and 100° C are 0 and 10 mm on a given fixed scale (as shown in fig.) coefficient of volume expansion this liquid varies with temperature as $\gamma = \gamma_0 \left(1 + \frac{T}{100}\right)$ (where T in °C)

Find the level (in mm) of liquid at 48°C.




**11.** A simple seconds pendulum is constructed out of a very thin string of thermal coefficient of linear expansion $\alpha = 20 \times 10^{-4}$ / $^{o}$C and a heavy particle attached to one end. The free end of the string is suspended from the ceiling of an elevator at rest. The pendulum keeps correct time at $0^{o}$C. When the temperature rises to $50^{o}$C, the elevator operator of mass 60kg being a student of Physics accelerates the elevator vertically, to have the pendulum correct time. Find the apparent weight (kgwt) of the operator when the pendulum keeps correct time at 50 $^{o}$C. (Take g = 10 m/s$^2$)

**12.** A steel rod 25 cm long has a cross-sectional area of 0.8 cm$^2$. The force required to stretch this rod by the same amount as the expansion produced by heating it through 10°C is 10 X. Find the value of X ? (Coefficient of linear expansion of steel is $10^{-5}$/°C and Young's modulus of steel is – $2 \times 10^{10}$ N/m$^2$.) **[JEE 1989, 3]**

**13.** A one liter flask contains some mercury. It is found that at different temperatures the volume of air inside the flask remains the same. The volume (in litre) of mercury in the flask is X/100. Find the value of X Coefficient of linear expansion of glass = $9 \times 10^{-6}$ / $^{o}$ C. Coefficient of volume expansion of mercury = $1.8 \times 10^{-4}$ /$^{o}$C. **[JEE 1991, 3]**

## PART - III : ONE OR MORE THAN ONE OPTIONS CORRECT TYPE

**1.** When two non reactive samples at different temperatures are mixed in an isolated container of negligible heat capacity the final temperature of the mixture can be :

(A) lesser than lower or greater than higher temperature

(B) equal to lower or higher temperature

(C) greater than lower but lesser than higher temperature

(D) average of lower and higher temperatures

**2.** When m gm of water at 10°C is mixed with m gm of ice at 0°C, which of the following statements are false?

(A) The temperature of the system will be given by the equation $m \times 80 + m \times 1 \times (T - 0) = m \times 1 \times (10 - T)$

(B) Whole of ice will melt and temperature will be more than 0°C but lesser than 10°C

(C) Whole of ice will melt and temperature will be 0°C

(D) Whole of ice will not melt and temperature will be 0°C

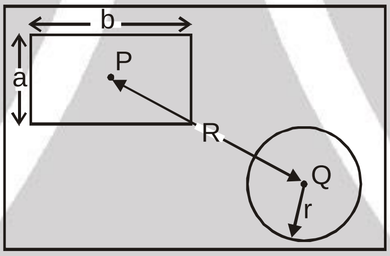
**3.** Two identical beakers with negligible thermal expansion are filled with water to the same level at 4°C. If one says A is heated while the other says B is cooled, then :

(A) water level in A must rise
(B) water level in B must rise
(C) water level in A must fall
(D) water level in B must fall

**4.** A bimetallic strip is formed out of two identical strips, one of copper and the other of brass. The coefficients of linear expansion of the two metals are $\alpha_C$ and $\alpha_B$. On heating, the temperature of the strips goes up by $\Delta T$ and the strip bends to form an arc of radius of curvature R. Then R is:

(A) Proportional to $\Delta T$
(B) inversely proportional to $\Delta T$
(C) proportional to $|\alpha_B - \alpha_C|$
(D) inversely proportional to $|\alpha_B - \alpha_C|$

**5.** There is a rectangular metal plate in which two cavities in the shape of rectangle and circle are made, as shown with dimensions. P and Q are the centres of these cavities. On heating the plate, which of the following quantities increase ?




(A) $\pi r^2$
(B) ab
(C) R
(D) b

**6.** A metallic wire of length $\ell$ is held between two supports under some tension. The wire is cooled through $\theta°$. Let Y be the Young's modulus, $\rho$ the density and $\alpha$ the thermal coefficient of linear expansion of the material of the wire. Therefore, the frequency of oscillations of the wire varies as

**[OLYMPIAD-2016_STAGE-1]**

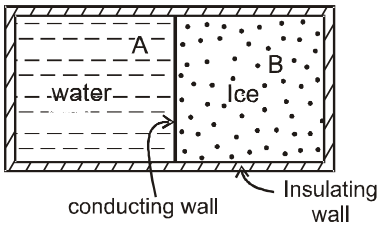
(A) $\sqrt{Y}$
(B) $\sqrt{\theta}$
(C) $\frac{1}{\ell}$
(D) $\sqrt{\frac{\alpha}{\rho}}$

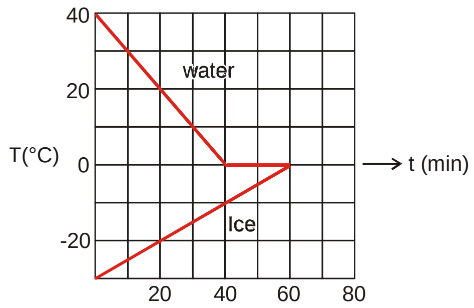
## PART - IV : COMPREHENSION

**Comprehension-1**

A 0.60 kg sample of water and a sample of ice are placed in two compartments A and B that are separated by a conducting wall, in a thermally insulated container. The rate of heat transfer from the water to the ice though the conducting wall is constant P, until thermal equilibrium is reached. The temperature T of the liquid water and the ice are given in graph as functions of time t. Temperature of the compartments remain homogeneous during whole heat transfer process.

Given specific heat of ice = 2100 J/kg-K
Given specific heat of water = 4200 J/kg-K
Latent heat of fusion of ice = $3.3 \times 10^5$ J/kg




**1.** The value of rate P is
(A) 42.0 W (B) 36.0 W (C) 21.0 W (D) 18.0 W

**2.** The initial mass of the ice in the container is equal to
(A) 0.36 kg (B) 1.2 kg (C) 2.4 kg (D) 3.6 kg

**3.** The mass of the ice formed due to conversion from the water till thermal equilibrium is reached, is equal to
(A) 0.12 kg (B) 0.15 kg (C) 0.25 kg (D) 0.40 kg

**Comprehension-2**

In a container of negligible heat capacity, 200 gm ice at 0°C and 100 gm steam at 100°C are added to 200 gm of water that has temperature 55°C. Assume no heat is lost to the surroundings and the pressure in the container is constant 1.0 atm. (Latent heat of fusion of ice = 80 cal/gm, Latent heat of vaporization of water = 540 cal/gm, Specific heat capacity of ice = 0.5 cal/gm-K, Specific heat capacity of water = 1 cal/gm-K)

**4.** What is the final temperature of the system ?
(A) 48°C (B) 72°C (C) 94°C (D) 100°C

**5.** At the final temperature, mass of the total water present in the system, is
(A) 472.6 gm (B) 483.3 gm (C) 493.6 gm (D) 500 gm

**6.** Amount of the steam left in the system, is equal to
(A) 16.7 gm (B) 12.0 gm (C) 8.4 gm
(D) 0 gm, as there is no steam left.

# Exercise-3

**Marked Questions can be used as Revision Questions.**

*** Marked Questions may have more than one correct option.**

## PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

**1.** A cube of coefficient of linear expansion $\alpha_s$ is floating in a bath containing a liquid of coefficient of volume expansion $\gamma_L$. When the temperature is raised by $\Delta T$, the depth upto which the cube is submerged in the liquid remains the same. Find the relation between $\alpha_s$ and $\gamma_L$ showing all the steps.
**[JEE-2004 (Mains), 2/60]**

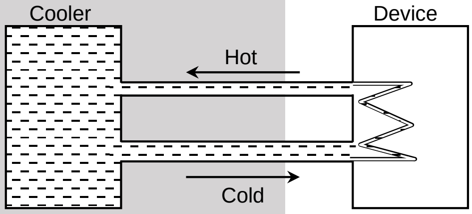
**2.** 2 liters water at 27°C is heated by a 1 kW heater in an open container. On an average heat is lost to surroundings at the rate 160 J/s. The time required for the temperature to reach 77°C is
**[JEE-2005 (Scr.), 3/84, –1]**
(A) 8 min 20 sec (B) 10 min (C) 7 min (D) 14 min

**3.** In an insulated vessel, 0.05 kg steam at 373 K and 0.45 kg of ice at 253 K are mixed. Find the final temperature of the mixture (in Kelvin). **[JEE-2006, 6/184, –1]**
Given, $L_{fusion}$ = 80 cal/gm = 336 J/gm, $L_{vaporization}$ = 540 cal/gm = 2268 J/gm,
$S_{ice}$ = 2100 J/kg K = 0.5 cal/gm K and $S_{water}$ = 4200 J/kg K = 1 cal/gmK

**4.** A piece of ice (heat capacity = 2100 J $kg^{-1}$ $^oC^{-1}$ and latent heat = 3.36 × $10^5$ J $kg^{-1}$) of mass m grams is at –5 ºC at atmospheric pressure. It is given 420 J of heat so that the ice starts melting. Finally when the ice-water mixture is in equilibrium, it is found that 1 gm of ice has melted. Assuming there is no other heat exchange in the process, the value of m is : **[JEE-2010, 3/163]**

**5.** Steel wire of length 'L' at 40°C is suspended from the ceiling and then a mass 'm' is hung from its free end. The wire is cooled down from 40°C to 30°C to regain its original length 'L'. The coefficient of linear thermal expansion of the steel is $10^{-5}$ /°C, Young's modulus of steel is $10^{11}$ N/$m^2$ and radius of the wire is 1 mm. Assume that L >> diameter of the wire. Then the value of 'm' in kg is nearly.
**[JEE-2011, 4/160]**

**6.** A water cooler of storage capacity 120 litres can cool water at a constant rate of P watts. In a closed circulation system (as shown schematically in the figure), the water from the cooler is used to cool an external device that generates constantly 3 kW of heat (thermal load). The temperature of water fed into the device cannot exceed 30°C and the entire stored 120 litres of water is initially cooled to 10°C. The entire system is thermally insulated. The minimum value of P (in watts) for which the device can be operated for 3 hours is : (Specific heat of water is 4.2 kJ $kg^{-1}$ $K^{-1}$ and the density of water is 1000 kg $m^{-3}$) **[JEE (Advanced) 2016; P-1, 3/62, –1]**




(A) 1600 (B) 2067 (C) 2533 (D) 3933

**7.** The ends Q and R of two thin wires, PQ and RS, are soldered (joined) together. Initially each of the wires has a length of 1m at 10 °C. Now the end P is maintained at 10 °C, while the end S is heated and maintained at 400°C. The system is thermally insulated from its surroundings. If the thermal conductivity of wire PQ is twice that of the wire RS and the coefficient of linear thermal expansion of PQ is 1.2 × $10^{-5}$ $K^{-1}$, the change in length of the wire PQ is. **[JEE (Advanced) 2016; P-2, 3/62, –1]**
(A) 0.78 mm (B) 0.90 mm (C) 1.56 mm (D) 2.34 mm

## PART - II : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

**1.** Time taken by a 836 W heater to heat one liter of water from 10ºC to 40ºC is :
**[AIEEE 2004; 4/120, –1]**
(1) 50 s (2) 100 s (3) 150 s (4) 200 s

**2.** The specific heat capacity of a metal at low temperature (T) is given as : **[AIEEE 2011,11 May; 4/120, –1]**

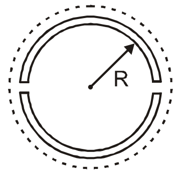
$$C_p\,(kJK^{-1}\,kg^{-1}) = 32\left(\frac{T}{400}\right)^3$$

A 100 gram vessel of this metal is to be cooled from 20K to 4K by a special refrigerator operating at room temperature (27ºC). The amount of work required to cool the vessel is :
(1) greater than 0.148 kJ (2) between 0.148 kJ and 0.028 kJ
(3) less than 0.028 kJ (4) equal to 0.002 kJ

**3.** A metal rod of Young's modulus Y and coefficient of thermal expansion $\alpha$ is held at its two ends such that its length remains invariant. If its temperature is raised by tºC, the linear stress developed in its is :
**[AIEEE 2011, 11 May; 4/120, –1]**
(1) $\frac{Y}{\alpha t}$ (2) $Y\alpha t$ (3) $\frac{1}{(Y\alpha t)}$ (4) $\frac{\alpha t}{Y}$

**4.** An aluminium sphere of 20 cm diameter is heated from 0ºC to 100ºC. Its volume changes by (given that coefficient of linear expansion for aluminium $\alpha_{Al}$ = 23 × $10^{-6}$/ºC) **[AIEEE 2011, 11 May; 4/120, –1]**
(1) 2.89 cc (2) 9.28 cc (3) 49.8 cc (4) 28.9 cc

**5.** A wooden wheel of radius R is made of two semicircular parts (see figure). The two parts are held together by a ring made of a metal strip of cross sectional area S and length L. L is slightly less than $2\pi R$. To fit the ring on the wheel, it is heated so that its temperature rises by $\Delta T$ and it just steps over the wheel. As it cools down to surrounding temperature, it presses the semicircular parts together. If the coefficient of linear expansion of the metal is $\alpha$, and its Young's modulus is Y, the force that one part of the wheel applies on the other part is : **[AIEEE 2012 ; 4/120, –1]**




(1) $2\pi SY\alpha\Delta T$ (2) $SY\alpha\Delta T$ (3) $\pi SY\alpha\Delta T$ (4) $2SY\alpha\Delta T$

**6.** A pendulum clock lose 12 s a day if the temperature is 40°C and gains 4 s a day if the temperature is 20°C. The temperature at which the clock will show correct time, and the co-efficient of linear expansion ($\alpha$) of the metal of the pendulum shaft are respectively : **[JEE (Main) 2016, 4/120, –1]**

(1) 60°C ; $\alpha = 1.85 \times 10^{-4}$/°C (2) 30°C ; $\alpha = 1.85 \times 10^{-3}$/°C
(3) 55°C ; $\alpha = 1.85 \times 10^{-2}$/°C (4) 25°C ; $\alpha = 1.85 \times 10^{-5}$/°C

**7.** A copper ball of mass 100 gm is at a temperature T. It is dropped in a copper calorimeter of mass 100 gm, filled with 170 gm of water at room temperature. Subsequently, the temperature of the system is found to be 75º C. T is given by : (Given : room temperature = 30ºC, specific heat of copper = 0.1 cal/gmºC) **[JEE (Main) 2017, 4/120, –1]**

(1) 825º C (2) 800ºC (3) 885ºC (4) 1250ºC

**8.** An external pressure P is applied on a cube at 0°C so that it is equally compressed from all sides. K is the bulk modulus of the material of the cube and $\alpha$ is its coefficient of linear expansion. Suppose we want to bring the cube to its original size by heating. The temperature should be raised by : **[JEE (Main) 2017, 4/120, –1]**

(1) $3PK\alpha$ (2) $\frac{P}{3\alpha K}$ (3) $\frac{P}{\alpha K}$ (4) $\frac{3\alpha}{PK}$

# Answers

## EXERCISE-1

### PART-I

**Section (A)**

**A-1.** H = 590 kcal **A-2.** 136 km

**A-3.** $\frac{3}{350} = 8.6 \times 10^{-3}$ ºC

**A-4.** (a) 100 J (b) 0 (c) 1/40 ºC

**A-5** 0ºC

**Section (B)**

**B-1.** %R < %A < %V

**B-2.** We will cool the system.

**B-3.** (a) $R_1' = R_1(1 + \alpha\Delta\theta)$ (b) $R_2' = R_2(1 + \alpha\Delta\theta)$

(c) $R_2' - R_1' = (R_2 - R_1)(1 + \alpha\Delta\theta)$

(d) $A' = (\pi R_2'^2 - \pi R_1'^2)(1 + 2\alpha\Delta\theta) = A(1 + 2\alpha\Delta\theta)$

**B-4.** (i) hollow sphere > solid sphere
(ii) hollow sphere = solid sphere

**B-5.** 0.75m **B-6.** $1.1 \times 10^{-2}$

**Section (C)**

**C-1.** (a) All tie (b) 50ºX, 50ºY, 50ºW.

**C-2 .** – 40ºC or – 40ºF

### PART-II

**Section (A)**

| | | | | | |
|---|---|---|---|---|---|
| **A-1.** | (D) | **A-2.** | (A) | **A-3.** | (A) |
| **A-4.** | (A) | | | | |

**Section (B)**

| | | | | | |
|---|---|---|---|---|---|
| **B-1.** | (A) | **B-2.** | (C) | **B-3.** | (C) |
| **B-4.** | (A) | **B-5.** | (C) | **B-6.** | (A) |

**Section (C)**

**C-1.** (A)

### PART-III

**1.** (A) – (p) ; (B) – (r) ; (C) – (s) ; (D) – (q)

**2.** (A) – (r) ; (B) – (q) ; (C) – (p) ; (D) – (s)

## EXERCISE-2

### PART-I

| | | | | | |
|---|---|---|---|---|---|
| **1.** | (C) | **2.** | (C) | **3.** | (A) |
| **4.** | (C) | **5.** | (C) | **6.** | (C) |
| **7.** | (B) | **8.** | (A) | **9.** | (A) |
| **10.** | (A) | **11.** | (C) | **12.** | (D) |
| **13.** | (D) | | | | |

### PART-II

| | | | | | |
|---|---|---|---|---|---|
| **1.** | 6 | **2.** | 3 | **3.** | 8 |
| **4.** | 8 | **5.** | 33 | **6.** | 53 |
| **7.** | 12 | **8.** | 42 | **9.** | 8 |
| **10.** | 4 | **11.** | 66 | **12.** | 16 |
| **13.** | 15 | | | | |

### PART-III

| | | | | | |
|---|---|---|---|---|---|
| **1.** | (BCD) | **2.** | (ABC) | **3.** | (AB) |
| **4.** | (BD) | **5.** | (ABCD) | **6.** | (ABCD) |

### PART-IV

| | | | | | |
|---|---|---|---|---|---|
| **1.** | (A) | **2.** | (C) | **3.** | (B) |
| **4.** | (D) | **5.** | (B) | **6.** | (A) |

## EXERCISE-3

### PART-I

| | | | | | |
|---|---|---|---|---|---|
| **1.** | $\gamma_L = 2\alpha_s$ . | **2.** | (A) | **3.** | 273 K |
| **4.** | 8 gm | **5.** | 3 | **6.** | (B) |
| **7.** | (A) | | | | |

### PART-II

| | | | | | |
|---|---|---|---|---|---|
| **1.** | (3) | **2.** | (2) | **3.** | (2) |
| **4.** | (4) | **5.** | (4) | **6.** | (4) |
| **7.** | (3) | **8.** | (2) | | |

# High Level Problems (HLP)

## SUBJECTIVE QUESTIONS

**1.** A thermally insulated, closed copper vessel contains water at 15ºC. When the vessel is shaken vigorously for 15 minutes, the temperature rises to 17ºC. The mass of the vessel is 100g and that of the water is 200g. The specific heat capacities of copper and water are 420 J/kg-K and 4200 J/kg-K respectively. Neglect any thermal expansion. (a) How much heat is transferred to the liquid-vessel system? (b) How much work has been done on this system? (c) How much is the increase in internal energy of the system?

**2.** The time represented by the clock hands of a pendulum clock depends on the number of oscillations performed by pendulum. Every time it reaches to its extreme position the second hand of the clock advances by one second that means second hand moves by two second when one oscillation is completed.

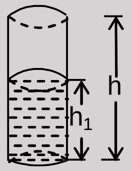
(a) How many number of oscillations completed by pendulum of clock in 15 minutes at calibrated temperature 20ºC

(b) How many number of oscillations are completed by a pendulum of clock in 15 minutes at temperature of 40ºC if $\alpha = 2 \times 10^{-5}$ /ºC

(c) What time is represented by the pendulum clock at 40ºC after 15 minutes if the initial time shown by the clock is 12 : 00 pm ?

(d) If the clock gains two seconds in 15 minutes in correct clock then find –
(i) Number of extra oscillations (ii) New time period (iii) change in temperature.

**3.** Consider a cylindrical container of cross section area 'A', length 'h' having coefficient of linear expansion $\alpha_C$. The container is filled by liquid of volume expansion coefficient $\gamma_L$ up to height $h_1$. When temperature of the system is increased by $\Delta\theta$ then




(a) Find out new height, area and volume of cylindrical container and new volume of liquid.

(b) Find the height of liquid level when expansion of container is neglected.

(c) Find the relation between $\gamma_L$ and $\alpha_c$ for which volume of container above the liquid level.
(i) increases (ii) decreases (iii) remains constant.

(d) If $\gamma_\ell > 3\alpha_c$ and $h = h_1$ then calculate, the volume of liquid overflow.

(e) If the surface of a cylindrical container is marked with numbers for the measurement of liquid level of liquid filled inside it. Assuming correct marking at initial temperature if we increase the temperature of the system by $\Delta\theta$ then
(i) Find height of liquid level as shown by the scale on the vessel. Neglect expansion of liquid
(ii) Find height of liquid level as shown by the scale on the vessel. Neglect expansion of container.
(iii) Find relation between $\gamma_L$ and $\alpha_C$ so that height of liquid level with respect to ground
(1) increases (2) decreases (3) remains constant.

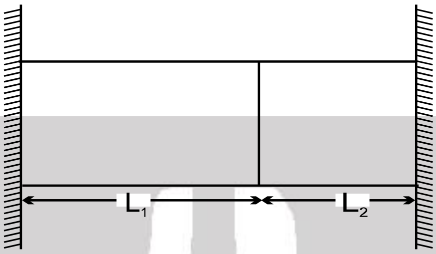
**4.** One gram of water (volume = 1 $cm^3$) becomes 1671 $cm^3$ of steam when boiled at a pressure of one atmosphere. Latent heat of vaporization at this pressure is 539 cal/gm. Compute the work done.
[1 atm = $1.013 \times 10^5$ $Nm^{-2}$] **[REE 1986, 3]**

**5.** A metal piece weighing 15g is heated to 100ºC and then immersed in a mixture of ice and water at the thermal equilibrium. The volume of the mixture is found to be reduced by 0.15 $cm^3$ with the temperature of mixture remaining constant. Find the specific heat of the metal. Given specific gravity of ice = 0.92, latent heat of fusion of ice = 80 cal/gm.

**6.** The brass scale of a barometer gives correct reading at 0º C. Coefficient of thermal expansion of brass is 0.00002/º C. The barometer reads 75 cm at 27º C. What is the correct atmospheric pressure at 27ºC? **[JEE 1989, 2]**

**7.** A clock with an iron pendulum keeps correct time at 20º C. How much will it lose or gain in a day if the temperature changes to 40º C? (Coefficient of cubical expansion of iron = 0.000036/º C) **[JEE1990, 3]**

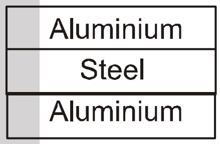
**8.** Two rods of different metals having same area of cross section A are placed end to end between two massive platforms, as shown in the figure. The first rod has a length $L_1$, coefficient of linear expansion $\alpha_1$ and Young's modulus $Y_1$. The corresponding quantities for the second rod are $L_2$, $\alpha_2$, and $Y_2$. The temperature of both the rods is now increased by Tº C. Find the force with which the rods act on each other ( at the higher temperature) in terms of given quantities. Also find the lengths of the rods at the higher temperature. Assume that there is no change in the cross sectional area of the rods and that the rods do not bend. There is no deformation of the walls. **[JEE 1990, 5]**




**9.** A composite rod is made by joining a copper rod end to end with a second rod of different material but of the same cross section. At 25º C the composite rod is 1 m in length of which the length of the copper rod is 30 cm. At 125º C the length of the composite rod increases by 1.91 mm. When the composite rod is not allowed to expand by holding it between two rigid walls it is found that the length of the two constituents do not change with the rise of temperature. Find the Young's modulus and the linear expansion of the second rod given that Young's modulus of for copper = $1.3 \times 10^{11}$ N/m$^2$ and the coefficient of linear expansion of copper = $1.7 \times 10^{-5}$ /º C. **[JEE 1990, 4]**

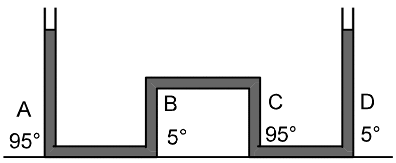
**10.** A piece of metal weighs 46 g in air. When it is immersed in a liquid of specific gravity 1.24 at 27º C it weighs 30 g. When the temperature of liquid is raised to 42º C the metal piece weighs 30.5 g. Specific gravity of liquid at 42º C is 1.20. Calculate the coefficient of linear expansion of the metal. **[JEE 1991, 3]**

**11.** Two Aluminium rods and a steel rod of equal cross-sectional area and equal length $\ell_0$ are joined rigidly side by side as shown in figure. Initially the rods are at 0ºC. Find the length of the rod at the temperature $\theta$ if young's modulus of elasticity of the aluminium and steel are $Y_a$ and $Y_s$ respectively and coefficient of linear expansion of aluminium and steel are $\alpha_a$ and $\alpha_s$ respectively.




**12.** Consider a metal scale of length 30 cm and an object. The scale is calibrated for temp 20ºC.
(a) What is the actual length of division which is shown as 1 cm by scale at 40ºC. Given $\alpha_s = 2\times10^{-5}$ /ºC.
(b) What will be the reading of scale at 40ºC if the actual length of object is 10 cm.
(c) What will be the actual length of object at 40 ºC if its measured length is 10 cm.
(d) What is % error in measurement for part (b) and (c).
(e) If the linear expansion coefficient of object is $\alpha_0 = 4 \times 10^{-5}$ and neglecting the expansion of scale, then answers of (b) and (c) parts.
(f) If $\alpha_0 = 4 \times 10^{-5}$ and $\alpha_s = 2 \times 10^{-5}$ then find answers of (b) and (c) part.

**13.** The apparatus shown in the figure consists of four glass columns connected by horizontal sections. The height of two central columns B & C are 49 cm each. The two outer columns A & D are open to the atmosphere. A & C are maintained at a temperature of 95º C while the columns B & D are maintained at 5º C. The height of the liquid in A & D measured from the base line are 52.8 cm & 51 cm respectively. Determine the coefficient of thermal expansion of the liquid. **[JEE 1997, 5]**



## HLP Answers

**1.** (a) zero (b) 1764 J (c) 1764 J

**2.** (a) 450 (b) 449 (c) 12:14:59 pm

(d) (i) 1 (ii) $\frac{900}{451}$ s (iii) $\frac{10^5}{450}$ ºC

**3.** (a) $h_f = h\{1+\alpha_c\,\Delta\theta\}$
$A_f = A\{1+2\alpha_c\,\Delta\theta\}$
$v_f = Ah\{1+3\alpha_c\,\Delta\theta\}$
volume of liquid $V_W = Ah_1(1+\gamma_L\,\Delta\theta)$

(b) $h_f = h_1\{1+\gamma_L\,\Delta\theta\}$

(c) (i) $3h\,\alpha_c > h_1\,\gamma_\ell$ (ii) $3h\,\alpha_c < h_1\,\gamma_\ell$
(iii) $3h\,\alpha_c = h_1\,\gamma_\ell$

(d) $\Delta V = Ah(\gamma_L - 3\alpha_C)\Delta\theta$

(e) (i) $h_f = h_1(1-3\,\alpha_C\,\Delta\theta)$
(ii) $h_f = h_1(1+\gamma_L\,\Delta\theta)$
(iii) (1) $\gamma_L > 2\alpha_c$ (2) $\gamma_L < 2\alpha_c$
(3) $\gamma_L = 2\alpha_c$

**4.** 169.171 J

**5.** 0.092 cal/gmºC

**6.** 75.0405 cm

**7.** 10.368 s

**8.** $F = \frac{AT(L_1\alpha_1+L_2\alpha_2)Y_1Y_2}{L_1Y_2+L_2Y_1}$, Length of the first rod = $L_1 + \frac{L_1L_2T(Y_1\alpha_1-Y_2\alpha_2)}{L_1Y_2+L_2Y_1}$,

Length of the second rod

$= L_2 + \frac{L_1L_2T(Y_2\alpha_2-Y_1\alpha_1)}{L_1Y_2+L_2Y_1}$

**9.** $Y_2 = 1.105 \times 10^{11}$ N/m$^2$, $\alpha_2 = 2\times10^{-5}$ /º C]

**10.** $\alpha = \frac{1}{43200}$ /º C = $2.31\times10^{-5}$ /º C

**11.** $\ell_0\left[1+\frac{2Y_a\alpha_a+Y_s\alpha_s}{2Y_a+Y_s}\theta\right]$

**12.** (a) $\ell = 1\{1+2\times10^{-5}\times20\}$

(b) $\ell = 10\{1-4\times10^{-4}\}$

(c) $\ell = 10\{1+4\times10^{-4}\}$

(d) $\%\ \ell_1 = -4\times10^{-2}\ \%$

$\%\ \ell_2 = \frac{-4\times10^{-2}}{1+4\times10^{-4}}\ \% \approx -4\times10^{-2}\ \%$

(e) $\ell_1 = 10\{1+20\times4\times10^{-5}\}$
$\ell_2 = 10\{1-20\times4\times10^{-5}\}$

(f) $\ell_1 = 10\{1+40\times10^{-5}\}$
$\ell_2 = 10\{1-40\times10^{-5}\}$

**13.** $\gamma = 2\times10^{-4}$ / °C

# HINT & SOLUTION OF CALORIMETRY & THERMAL EXPANSION
## EXERCISE-1
## PART-1

**A 1.** $H = 1 \times 540 + 1 \times \frac{1}{2} \times 100 = 590$ kcal.

**A 2.** $H = m \times 3.4 \times 10^5$ J

$E = m \times 10 \times h \times \frac{1}{4}$ J

$m \times 3.4 \times 10^5 = m \times 10 \times \frac{h}{4} = h = 136$ km.

**A 3.** mg sin37ºd = m s $\Delta\theta$

$\Delta\theta = \frac{10\times3\times0.6}{5\times420} = \frac{3\times6}{5\times420} = \frac{3}{350} = 8.6 \times 10^{-3}$ ºC.

**A 4.** (a) W = mgh = 10 × 10 × 1 = 100 J
(b) No heat supplied from out side
(c) $100 = 4000\ \Delta\theta$
$\Delta\theta = \frac{1}{40}$ ºC

**A 5** Heat liberated when 300 g water at 25ºC goes to water at 0ºC :
$Q = ms\ \Delta\theta$
$= (300)\ (1)\ (25)$
$= 7500$ cal
From Q = mL, this much heat can melt mass of ice given by
$m = \frac{Q}{L} = \frac{7500}{80}$
$= 93.75$ g
i.e., whole ice will not melt.
Hence, the mixture will be at 0ºC
Mass of water in mixture = 300 + 93.75 = 393.75 g and
Mass of ice in mixture = 100 – 93.75 = 6.25 g

**B 1.** $\Delta R\% = \alpha\Delta T$
$\Delta A\% = 2\alpha\Delta T$
$\Delta V\% = 3\alpha\Delta T$
% R < % A < % V

**B 2.** Since $\alpha_s < \alpha_b$ there will be less contraction in steel as compared to brass.

**B 3.** **Ans.** (a) $R_1' = R_1 (1 + \alpha\Delta\theta)$
(b) $R_2' = R_2 (1 + \alpha\Delta\theta)$
(c) $R_2' - R_1' = (R_2 - R_1)(1 + \alpha\Delta\theta)$
(d) $A' = (\pi R_2^2 - \pi R_1'^2)(1 + 2\alpha\Delta\theta) = A(1 + 2\alpha\Delta\theta)$

**B 4.** When hollow sphere is filled air, expansion will be due to pressure of the gas also.

**B 5.** $\ell_2 - \ell_1 = 0.25$

$\ell_1\alpha_1 = \ell_2\alpha_2$

$\ell_1 \times 22 \times 10^{-6} = \ell_2 \times 11 \times 10^{-6}$ $\quad 2\ell_1 = \ell_2$

$\ell_1 = 0.25$ m

$\ell_2 = 0.50$ m $\Rightarrow$ $\ell_1 + \ell_2 = 0.75$ m

**B 6** $100 \times \dfrac{\Delta\ell}{\ell_0} = \dfrac{\ell_0 \times 11 \times 10^{-6} \times 10}{\ell_0} \times 100$

$= 11 \times 10^{-3}$

$= 1.1 \times 10^{-2}$

**C 1.** (a) Since the term diff. between F.P. & B.P. is 90º in all the three scale so all are tie.

(b) $\dfrac{X-(-20)}{90} = \dfrac{W-30}{90} = \dfrac{Y-0}{90}$

$\dfrac{50+20}{90} = \dfrac{W-30}{90}$ $\Rightarrow$ W = 100 $\quad$ 50ºX = 100º W

$\dfrac{W-30}{90} = \dfrac{Y}{90}$ $\Rightarrow$ W = 80 $\quad$ 50ºY = 80ºW

So, 50º X, 50º Y, 50º W.

**C 2 .** Let at XºF = XºC

$\dfrac{X-32º}{180} = \dfrac{X-0}{100}$

10X – 320 = 18 X $\Rightarrow$ X = –40ºC

So, –40ºC or –40º F

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## PART - II

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**A 1.** $mc\theta = m_iL$ $\Rightarrow$ $m_i = \dfrac{mc\theta}{L}$

**A 2.** $\dfrac{m}{1000} \times 3.36 \times 10^5 = \left(\dfrac{100-m}{1000}\right) 2.1 \times 10^6$

$m \times 3.36 = 2100 - 21m$

$m = \dfrac{2100}{21+3.36} = 86.2$ g.

**A 3.** For minimum value of m, the final temperature of the mixture must be 0°C.

$\therefore 20 \times \dfrac{1}{2} \times 10 + 20 \times 80 = m\,540 + m.1.\,100$

$\therefore m = \dfrac{1700}{640} = \dfrac{85}{32}$ gm.

**A 4.** **(A)**

Heat released by 5 kg of water when its temperature falls from 20ºC to 0ºC is,

$Q_1 = ms\Delta\theta = (5)\,(10^3)\,(20 - 0) = 10^5$ cal

when 2 kg ice at –20ºC comes to a temperature of 0ºC, it takes an energy

$Q_2 = ms\Delta\theta = (2)\,(500)\,(20) = 0.2 \times 10^5$ cal

The remaining heat

$Q = Q_1 - Q_2 = 0.8 \times 10^5$ cal will melt mass m of the ice, where

$m = \dfrac{0.8\times10^5}{80\times10^3} = 1$ kg

So, the temperature of the mixture will be 0ºC,

mass of water in it is 5 + 1 = 6 kg and mass of ice is 2 – 1 = 1 kg.

**B 1.** On heating the expansion will take place hence both the distances will increase.

**B 2.** $6 \times 10^{-5} = 1 \times 12 \times 10^{-6} \times \Delta T$

$\frac{6\times10^{-5}}{12\times10^{-6}} = \Delta T \Rightarrow \Delta T = 5ºC.$

**B 3.** On heating volume of substance increases while mass of the substance remains the same. Hence the density will decrease

**B 4.** In bimetallic strips the two metals have different thermal expansion coefficient. Hence on heating it bents towards the metal with lower thermal expansion coefficient.

**B 5.** Given $\Delta\ell_1 = \Delta\ell_2$

or $\ell_1 \alpha_a t = \ell_2 \alpha_s t$ $\therefore$ $\frac{\ell_1}{\ell_2} = \frac{\alpha_s}{\alpha_a}$

or $\frac{\ell_1}{\ell_1+\ell_2} = \frac{\alpha_s}{\alpha_a+\alpha_s}$

**B 6.** $V_\ell > V_e$

$\gamma > 3\alpha.$

**C 1.** $\frac{\Delta C}{100º} = \frac{\Delta F}{180º}$

$\frac{25}{100º} \times 180º = \Delta F \Rightarrow \Delta F = 45ºF.$

# PART - III

**1.** (A) Buoyant force = Mg = constant = $V_{sub}dg$ $\Rightarrow$ $V_{sub} = \frac{Mg}{dg}$.

volume of displace fluid = constant

$\therefore$ density of fluid must be constant.

(B) $(V_{solid} - V_{sub})$ = constant

$\Rightarrow \left(V_{solid} - \frac{M_{solid}}{\rho_{liquid}}\right)$ = constant $V \times 3\alpha \times \Delta T = \frac{M\gamma\Delta T}{d} \Rightarrow \gamma = 3\alpha\frac{d}{\rho}$

(C) $Ah_{in}\, d_{liquid} = A(h_{in} + h_{out})\, \rho_{solid} = M$ (mass of solid)

$h_{out} = \frac{M}{A\rho_{solid}} - h_{in} = \frac{M}{A\rho_{solid}} - \frac{M}{Ad_{liquid}}$ = constant

$$\frac{M(1+3\alpha\Delta T)}{A(1+2\alpha\Delta T)\ \rho} - \frac{M\ (1+\gamma\Delta T)}{A(1+2\alpha\Delta T)\ d} = \frac{M}{A\rho} - \frac{M}{Ad}$$

$\gamma = 2\alpha + \alpha\frac{d}{\rho}$

(D) $Ah_{in}\, d_{liquid}\, g$ = Buoyant force = constant = Mg

$A_0(1 + 2\alpha\Delta T)\, h_{in} \frac{d}{1+\gamma\Delta T}$ = constant

$h_{in} = \frac{M}{A_0 d}(1 + (\gamma - 2\alpha)\Delta T)$

$\gamma = 2\alpha$

**2.** **Ans.** (A) – (r) ; (B) – (q) ; (C) – (p) ; (D) – (s)

# EXERCISE-2
## PART - I

**1.** $V \times 4.5 \times 10^3 \times 0.1 \times T = \frac{V}{2} \times 0.9 \times 10^3 \times 80$

$T = \frac{9 \times 80}{2 \times 4.5} = 80ºC.$

**2.** $Pt = ML$

$L = \frac{Pt}{M}$

**3.** Heat released by steam = heat gained by water and calorimetry

$mL + m \times S\ (100 - 80) = (1.1 + 0.02) \times S \times (80 - 15)$

$m \times 540 + 20\ m = 1.12 \times 65$

$m = 0.130$ kg.

**4.** $I = MK^2$

$I' = MK^2(1 + 2\alpha\Delta T)$

$\Delta I = I\ 2\alpha\ \Delta T$

**5.** $\ell_1(1 + \alpha_1\Delta T) + \ell_2(1 + \alpha_2\ \Delta T) = \ell_f$

$\ell_f = \ell_1 + \ell_2 + (\ell_1\alpha_1 + \ell_2\alpha_2)\Delta T$

$\ell_f = (\ell_1 + \ell_2)\left(1 + \frac{\ell_1\alpha_1 + \ell_2\alpha_2}{\ell_1 + \ell_2}\Delta T\right).$

**6.** $P\Delta V = nR\Delta T \qquad \Delta V = \frac{nR}{P}\Delta T$

$\Delta V = \frac{V}{T}\ \Delta T \qquad$ So, $\gamma = \frac{1}{T}$

**7.** $w_1 = Mg - F_B$

$w_2 = Mg - F_B\left[\frac{1 + \gamma_m\Delta T}{1 + \gamma_\ell\Delta T}\right] = Mg - F_B[1 + (\gamma_m - \gamma_\ell)\Delta T]$

Since, $\gamma_m < \gamma_\ell$ So, $w_2 > w_1$

**8.** For floating condition

$mg = B$

$v\ d_0 g = v_{in}\ d_L g$

where $d_0$ = density of object

$d_L$ = density of liquid

fraction of volume submerged in liquid ($f_1$) = $\frac{v_{in}}{v}$

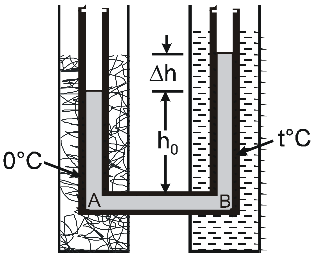
$f_1 = \frac{d_0}{d_L}$

after increasing temp, by $\Delta T$

$d_0' = \frac{d_0}{1 + \gamma_1\Delta T} \qquad d_L' = \frac{d_L}{1 + \gamma_2\Delta T} \qquad f_1' = \frac{d_0'}{d_L'} = \frac{d_0}{d_L} \times \frac{1 + \gamma_2\Delta T}{1 + \gamma_1\Delta T}$

$\therefore \qquad \frac{f_1'}{f_1} = \frac{1 + \gamma_2\Delta T}{1 + \gamma_1\Delta T}$ **Ans.**

**9.**




In equilibrium,
Pressure at A = Pressure at B

$h_0 \rho_0 g = (h_0 + \Delta h)\ \dfrac{\rho_0}{1+\gamma t} g \Rightarrow \quad \gamma = \dfrac{\Delta h}{h_0 t}$ **Ans.**

**10.**


$$\frac{KA(10)}{x} = L\frac{d}{dt}(\rho A x)$$

$$\frac{2.2\times(10)}{x} = 3.4\times10^5\times0.9\times10^3\,\frac{dx}{dt}$$

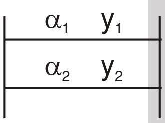
$$\frac{22}{3.4\times0.9\times10^8}\int_0^t dt = \int_0^{0.5} x\,dx$$

$$\frac{22}{306\times10^6}t = \frac{1}{2}(0.5)^2$$

$$t = \frac{(0.5)^2\times306\times10^6}{44}\text{sec} \quad = \frac{306}{44\times4}\times10^6\text{ sec}$$

$$= \frac{306000\times10^3}{44\times4\times24\times3600} \approx 20\text{ days}$$

**11.**




$$\frac{\alpha_1}{\alpha_2} = \frac{2}{6}$$

$\because\ \dfrac{F}{A} = Y\,\alpha\,\Delta\theta \qquad \because\ \Delta\theta$ is same for both

$$\frac{\frac{F_1}{A_1}}{\frac{F_2}{A_2}} = \frac{Y_1\alpha_1}{Y_2\alpha_2}$$

$$\frac{Y_1}{Y_2} = \frac{\alpha_2}{\alpha_1} = 3 : 1$$

**12.** $R = \frac{d}{(\Delta\alpha)t} = \frac{2x10^{-3}}{(20x10^{-6})(50)} = 2\text{ m}$

**13.** $dl_{eq} = dl_1 + dl_2$

$(l_1 + l_2)\alpha_{eq}dT = l_1\alpha_1 dT + l_2\alpha_2 dT$

$\alpha_{eq} = \frac{l_1\alpha_1 + l_2\alpha_2}{l_1 + l_2}$

## PART - II

**1.** $20 \times 4200 \times 5 = 5 \times 10^{-4} \times 2.27 \times 10^{6} \times t$

$t = \frac{20 \times 4200}{227 \times 60} = 6.16\text{ min}$

**2.** $t \times 59000 = 150 \times 4200 \times (38 - 21)$

$t = 3\text{ min}$

**3.** $dQ = mcdT$

$Q = \int dQ = \int mc\,dT$

$Q = \int 2\left(0.2 + 0.16T + 0.024T^2\right)dT$

$Q = 2\,[0.2 \times 5 + 0.08 \times 25 + 0.008 \times 125] = 2[1 + 2 + 1] = 8\text{ cal}$

**4.** $6000 = 50 \times 80 + 250 \times 1 \times \Delta\theta$

$6000 - 4000 = 250 \times \Delta\theta \Rightarrow \Delta\theta = \frac{2000}{250} = 8°C$

**5.** Energy given by the sun = $1400 \times 0.2 \times t$

Energy required to melt the ice = $280 \times 10^{-3} \times 3.3 \times 10^{5}$

$1400 \times 0.2 \times t = 280 \times 10^{-3} \times 3.3 \times 10^{5}$

$t = 330\text{ sec.}$

**6.** Heat released by bullet = heat gained by ice

$\Rightarrow \quad \frac{1}{2}mv^2 + m_1S\Delta\theta = mL$

$\Rightarrow \quad \frac{1}{2} \times (50 \times 10^{-3}) \times (840)^2 \times \frac{1}{4.2} + 50 \times 0.02 \times (30 - 0) = \frac{66 \times 100}{550}\ m \times 80$

$m = 52.875\text{ gm}$ **Ans.**

**7.** Heat released by steam = Heat absorbed by water

$m_1L + m_1 \times S\,(100 - 90) = m_2 \times S\,(90 - 24)$

$540\,m_1 + 10\,m_1 = 66\,m_2 \quad \Rightarrow \quad m_1 = 12\text{ gm}$

**8.** Energy supplied by coil = heat gain by water and calorimetry

$\Rightarrow \quad Pt = (m_1 + w) \times S\,\Delta\theta$

$\Rightarrow \quad 90 \times (10 \times 60) = (360 + 40) \times 4.2 \times (\theta - 10)$

$\theta = 42.14°C$ **Ans.**

**9.** New length of one half is $\ell = \ell_0/2\,(1 + \alpha\Delta\theta)$

$$\Rightarrow \left(\sqrt{\left(\frac{\ell_0}{2}\right)^2 + x^2}\right)^2 = \left(\frac{\ell_0}{2}\right)^2 (1 + \alpha\Delta\theta)^2$$

$$\Rightarrow x^2 = \left(\frac{\ell_0}{2}\right)^2 \times 2\alpha\Delta\theta \Rightarrow x = \frac{\ell_0}{2} \times \sqrt{2\alpha\Delta\theta}$$

$$\Rightarrow x = \frac{4}{2} \times \sqrt{25 \times 10^{-6} \times 32 \times 2} \Rightarrow x = 8 \text{ cm}$$

**10.** $dv = Adx$

$$v_0\,\gamma_0\left(\frac{T}{100} + 1\right) dT = Adx$$

$$\Rightarrow v_0\,\gamma_0 \int_0^{100} \left(\frac{T}{100} + 1\right) dT = A \int_0^{10} dx \Rightarrow v_0\,\gamma_0 \times \left(\frac{100^2}{2 \times 100} + 100\right) = A \times 10$$

$\Rightarrow v_0\,\gamma_0 \times 150 = A \times 10$ .....(I)

**II case**

$$\Rightarrow v_0\,\gamma_0 \int_0^{48} \left(\frac{T}{100} + 1\right) dT = A \int_0^{x} dx$$

$$v_0\,\gamma_0 \times \left(\frac{48^2}{2 \times 100} + 48\right) = A \times x \quad .....(II)$$

$$\frac{1}{15} \times 48\,(0.24 + 1) = x$$

$x = 1.24 \times 3.2 = 3.968 = 4$ mm

**11.** $T = 2\pi\sqrt{\dfrac{\ell}{g}}$

when the temperature is raised length changes to $\ell\,(1 + \alpha\,\Delta T)$

when the lift accelerates upwards

$g_{eff} = g + a$

new period of pendulum,

$$T' = 2\pi\sqrt{\frac{\ell(1+\alpha\Delta T)}{g+a}} \Rightarrow T' = T \Rightarrow g\frac{(1+\alpha\Delta T)}{(g+a)} = 1$$

or $a = g\,\alpha\,\Delta T = 10 \times 20 \times 10^{-4} \times 50 = 1$ m/sec$^2$

$w_{eff} = m\,(g + a) = 60\,(10 + 1) = 660$ N

**12.** $Y = \dfrac{F/A}{\Delta\ell/\ell}$

$\because \quad \dfrac{\Delta\ell}{\ell} = \alpha\,\Delta\theta$

$\therefore \quad F = Y\,\alpha\,A\,\Delta\theta$

$= 2 \times 10^{10} \times (0.8 \times 10^{-4}) \times 10^{-5} \times 10$

$F = 160$ N **Ans.**

**13.** $V_C - V_{Hg} = V_C' - V_{Hg}'$ = volume of air

$\Rightarrow \quad V_C' = V_C(1 + 3\alpha_s \Delta\theta)$

$V_{Hg}' = V_{Hg}(1 + \gamma_L \Delta\theta)$

So $\quad V_C \times 3\alpha_s = V_{Hg} \times \gamma_L$

$$V_{Hg} = \frac{1\times3\times9\times10^{-6}}{1.8\times10^{-4}}$$

$V_{Hg} = 0.15$ L **Ans.**

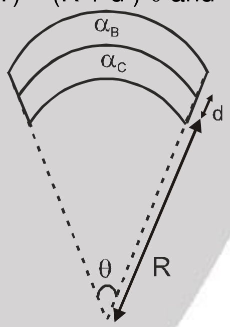
## PART - III

**1.** When two samples at different temperatures are mixed and there is a state change and temperature remains constant then the temperature of the mixture is equal to lower or higher temperature.

**2.** Heat required to melt ice = m × 80
Heat given by the water = m × 10 × 1 = 10m
Heat required to melt ice > Heat given by water so complete ice will not melt.

**3.** On heating or cooling water from 4ºC it expands in both cases.

**4.** Let $\ell_0$ be the initial length of each strip before heating.
Length after heating will be
$l_B = l_0(1 + \alpha_B \Delta T) = (R + d)\,\theta$ and $l_C = l_0(1 + \alpha_C \Delta T) = R\theta$




$$\therefore \quad \frac{R+d}{R} = \left(\frac{1+\alpha_B\Delta T}{1+\alpha_C\Delta T}\right)$$

$$\therefore \quad 1+\frac{d}{R} = 1+(\alpha_B-\alpha_C)\Delta T \quad \text{[ From binomial expansion ]}$$

$$\therefore \quad R = \frac{d}{(\alpha_B-\alpha_C)\Delta T}$$

or $\quad R \propto \dfrac{d}{\Delta T}$ and $\propto \dfrac{1}{|\alpha_B - \alpha_C|}$

**5.** On heating, every dimension increases.

**6.** $f = \dfrac{n}{2\ell}\sqrt{\dfrac{T}{\mu}}$ ; $\dfrac{T/A}{\Delta\ell/\ell} = \gamma$

$$\frac{T/A}{\ell\alpha\Delta\theta/\ell} = \gamma \Rightarrow T = A\alpha\gamma\theta \qquad \therefore\ f = \frac{1}{2\ell}\sqrt{\frac{A\alpha\gamma\theta\ell}{m}} = \frac{1}{2\ell}\sqrt{\frac{\alpha\gamma\theta}{\rho}}$$

# PART - IV

**1.** In 40 min. temperature of water has come down by 40°C.

Therefore rate $P = \frac{mS\Delta T}{t} = \frac{0.60 \times 4200 \times 40}{40 \times 60} = 42.0$ W

**2.** Sample of ice has been receiving heat at constant rate P from water. Its temperature has increased by 30°C in time 60 min.

Therefore $\frac{m_i s_i \Delta T_i}{P} = 60$ min. $\Rightarrow$ $m = \frac{(60 \times 60s) \times (42\ \ W)}{(2100 J/kg)\ \ .\ \ (30°C)} = 2.4$ kg

**3.** Thermal equilibrium reaches after 60 min. Ice conversion takes place for 20 min. During this time water at 0°C continues to give heat at rate P.

$m \times L_f = P \times (20 \times 60s) \Rightarrow m = \frac{42 \times 20 \times 60}{3.3 \times 10^5}$ kg = 0.15 kg

**4.** (D) 100°C

**5.** (B) 483.3 gm

**6.** **4 - 6**

As steam has comparatively large amount of heat to provide in the form of latent heat we check what amount of heat is required by the water and ice to go up to 100°C, that is

$(m_i L + m_i S_w \Delta T) + m_w . S_w . \Delta T$

$= [(200 \times 80) + (200 \times 1 \times 100)] + (200 \times 1 \times 45)$

= 45,000 cal.

That is given by m mass of steam, then

$m_s.L = 45,000$

$m_s = \frac{45,000}{540} = \frac{500}{6} = 83.3$ gm

therefore 83.3 gm steam converts into water of 100°C.

Total water = 200 + 200 + 83.3 = 483.3 gm

steam left = 16.7 gm.

# EXERCISE-3
## PART - I

**1.** When the temperature is increased, volume of the cube will increase while density of liquid will decrease. The depth upto which the cube is submerged in the liquid remains the same, hence the upthrust will not change.

F = F'

$\therefore \quad V_i\rho_L g = V'_i\, \rho'_L g \quad$ ($V_i$ = volume immersed)

$\therefore \quad (Ah_i)\,(\rho_L)\,(g) = (1 + 2\alpha_s\Delta T)\,(Ah_i)\left(\dfrac{\rho_L}{1+\gamma_L\Delta T}\right) g$

Solving this equation, we get $\gamma_L = 2\alpha_s$.

**2.** **(A)** Net heat given/sec = 1000 – 160

= 840 J/S

if it takes time t then

840 t = 2000 × 4.2 × (77 – 27)

t = 500 sec = 8 min 20 sec.

**3.** $\Sigma\Delta Q = 0$

Heat lost by steam to convert into 0ºC water

$H_L = 0.05 \times 540 + 0.05 \times 100 \times 1$

$= 27 + 5 = 32$ kcal

Heat required by ice to change into 0º C water

$H_g = 0.45 \times \dfrac{1}{2} \times 20 + 0.45 \times 80 = 4.5 + 36.00 = 40.5$ kcal

Thus, final temperature of mixture is 0ºC = 273 K

**4.** S = 2100 J $kg^{-1}$ º$C^{-1}$

L = $3.36 \times 10^5$ J $kg^{-1}$

$420 = m\,S\,\Delta\theta + (1) \times 10^{-3} \times L$

$420 = m\,S\,(5) + 3.36 \times 10^2$

$420 - 336 = m(2100) \times 5$

$m = \dfrac{1}{125} \times 1000 = 8$ gm.

**5.** $\dfrac{F}{A} = y\,\dfrac{\Delta L}{L}$

$\dfrac{mg}{A} = y \times (\alpha\Delta\theta)$

$m = \dfrac{Ay\alpha(\Delta\theta)}{g} = \dfrac{\pi r^2 y\alpha(\Delta\theta)}{g} = \dfrac{\pi(10^{-3})^2 \times 10^{11} \times 10^{-5} \times 10}{10} = \pi \;\; 3$

**Ans.** 3

**6.** Heat generated in device in 3 hours = $3 \times 3600 \times 3 \times 10^3 = 324 \times 10^5$ J

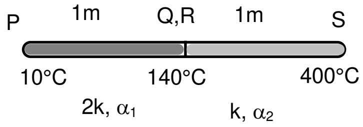
Heat used to heat water = $ms\Delta\theta = 120 \times 1 \times 4.2 \times 10^3 \times 20$ J

Heat absorbed by coollant = Pt = $324 \times 10^5 - 120 \times 1 \times 4.2 \times 10^3 \times 20$ J

Pt = $(325 - 100.8) \times 10^5$ J

$P = \dfrac{223.2 \times 10^5}{3600} = 2067$W

**7.** **(A)**




$d\ell = dx\alpha_1\,(\theta - 10)$

$\Delta\ell = \int d\ell \qquad \because \left(\dfrac{\theta - 10}{x}\right) = \dfrac{130}{1}$

$\Rightarrow \qquad \theta = 10 + 130x$

$\Delta\ell = \int_0^1 (130x)\alpha_1 dx \qquad \Delta\ell = 130\,\alpha_1\,\dfrac{x^2}{2}$

$\Delta\ell = 130 \times 1.2 \times 10^{-5} \times \dfrac{1}{2} = 78 \times 10^{-5} = 0.78\text{mm}$ **Ans. (A)**

## PART - II

**1.** Let time taken in boiling the water by the heater is t sec. Then

$Q = ms\Delta T$

$\dfrac{836}{4.2}t = 1 \times 1000\,(40^{\circ} - 10^{\circ})$

$\dfrac{836}{4.2}t = 1000 \times 30 \;\Rightarrow\; t = \dfrac{1000 \times 30 \times 4.2}{836} = 150 \text{ sec}$

**2.** $Q = \int mcdT$

$= \int_{20}^{4} 0.1 \times 32 \times \left(\dfrac{T^3}{400^3}\right) dT$

$\approx 0.002$ k J.

$W_{min} = \left(\dfrac{300}{20} - 1\right) \times 0.002$

$W_{max} = \left(\dfrac{300}{4} - 1\right) \times 0.002$

**3.** $\dfrac{\Delta\ell}{\ell} = \alpha.\Delta T$

and $Y = \dfrac{F/A}{\Delta\ell/\ell}$

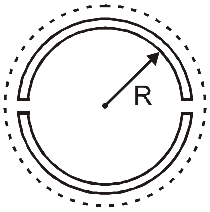
So, $F = AY\alpha t$

Thermal stress $\left(\dfrac{F}{A}\right) = Y\alpha t$.

**4.** $\Delta v = v_0(3\alpha)\,\Delta T = \dfrac{4}{3}\pi(10)^3 \times 3 \times 23 \times 10^{-6} \times 100$

$\Delta v = 28.9$ cc.

**5.**




$L \Rightarrow S$

$\Delta L = L\,\alpha\,\Delta T$

$\frac{F}{A} = \frac{\Delta L}{L} Y$

$F = \alpha \Delta T\, YS$

So, $T = 2F$

$T = 2\alpha\Delta T\, YS$

**6.** $\frac{12}{24 \times 3600} = \frac{1}{2}\alpha(40 - T)$ ...(i)

$\frac{-4}{24 \times 3600} = \frac{1}{2}\alpha(20 - T)$ ...(ii)

from equation (i) and (ii)

$-3 = \frac{40 - T}{20 - T}$

$-60 + 3T = 40 - T$

$4T = 100$

$T = 25$

from equation (ii)

$\frac{-4}{24 \times 3600} = \frac{1}{2}\alpha(20 - 25)$

$\frac{4}{24 \times 3600} = \frac{1}{2} \times \alpha \times 5$

$\alpha = \frac{8}{24 \times 3600 \times 5} = 1.85 \times 10^{-5}/°C$

**7.** Heat given by cupper ball = Heat taken by water + Heat taken by calorimeter system

(100) (0.1) (T – 75) = (170) (1) (75 – 30) + (100) (0.1) (75 – 30)

10T – 750 = 8100

10T = 8850

T = 885°C

**8.** $\Delta p = -K\frac{\Delta V}{V}$

$\frac{\Delta V}{V} = -\gamma\Delta\theta = -3\,\alpha\Delta\theta$

$3\alpha\Delta\theta K = p$

$\Delta\theta = \frac{p}{3\alpha K}$

## HIGH LEVEL PROBLEMS (HLP)
## SUBJECTIVE QUESTIONS

**1.** (a) Zero, because there is no heat transfer from outside.
(b) $W = 100 \times 10^{-3} \times 420 \times 2 + 200 \times 10^{-3} \times 4200 \times 2 = 1764$ J.
(c) $\Delta U = 1764$ J

**2.** (a) Number of oscillations = $\frac{15 \times 60}{2} = 450$

(b) $\Delta T = \frac{1}{2} \times \alpha \Delta\theta T = \frac{1}{2} \times 2 \times 10^{-5} \times 20 \times 2 = 4 \times 10^{-4}$ sec.

$T' = 2 + 4 \times 10^{-4} = 2.0004$ sec.

Number of oscillations = $\frac{15 \times 60}{2.0004} = 449.91$

Number of complete oscillations = 449
(c) Last oscillation will not be complete.
So, time shown = 12 : 15 : 00 – 0:00 : 01
= 12:14:59 pm

(d) $2 = \frac{1}{2} \times 2 \times 10^{-5} \times \Delta\theta \times 15 \times 60$

$\Rightarrow \quad \Delta\theta = \frac{2 \times 10^5}{15 \times 60} = \frac{10^5}{450}$ ºC

$\Delta T = \frac{1}{2}\alpha \ \Delta\theta \ T = \frac{1}{2} \times 2 \times 10^{-5} \times \frac{10^5}{450} \times 2 = \frac{2}{450}$

$T' = 2 - \frac{2}{450} = \frac{898}{450}$ sec.

Number of oscillations = $\frac{15 \times 60 \times 450}{898} = 451$

Number of extra oscillations = 451 – 450 = 1

**3.** (a) $h_f = h \{ 1 + \alpha_c \Delta\theta \}$
$A_f = A \{ 1 + 2\alpha_c \Delta\theta \}$
$v_f = Ah \{ 1 + 3\alpha_c \Delta\theta \}$
volume of liquid $V_W = Ah_1 ( 1 + \gamma_L \Delta\theta)$

(b) $h_f = h_1 \{ 1 + \gamma_L \Delta\theta \}$

(c) (i) $3h\,\alpha_c > h_1 \gamma_\ell$ (ii) $3h\,\alpha_c < h_1 \gamma_\ell$ (iii) $3h\,\alpha_c = h_1 \gamma_\ell$

(d) $\Delta V = Ah\,(\gamma_L - 3\alpha_C)\Delta\theta$

(e) (i) $h_f = h_1 (1 - 3\,\alpha_C \Delta\theta)$ (ii) $h_f = h_1 (1 + \gamma_L \Delta\theta)$
(iii) (1) $\gamma_L > 2\alpha_c$ (2) $\gamma_L < 2\alpha_c$ (3) $\gamma_L = 2\alpha_c$

**4.** Process is isobaric
$W = P(V_2 - V_1) = 1.013 \times 10^5 \times (1671 - 1) \times 10^{-6}$
W = 169.171 J **Ans.**

**5.** $\frac{m}{\rho_{ice}} - \frac{m}{\rho_{H_2O}} = 0.15$

$m = \frac{0.15 \times 0.92}{0.08}$

$\therefore \quad H_{lost} = H_{gained} \quad \Rightarrow \quad 15 \times S \times 100 = \frac{0.15 \times 0.92}{0.08} \times 80$

**S = 0.092 cal/gmºC.**

**6.** $h = h_0 (1 + \alpha\Delta\theta)$
$h = 75\{1 + 0.00002 \times 27\}$
$h = 75.0405$ cm **Ans.**

**7.** Gain or loss in time due to thermal expansion

$$\Delta t = \frac{1}{2} \times \Delta\theta \times t \times \alpha \quad \left(\alpha = \frac{0.000036}{3} /^{\circ}C\right)$$

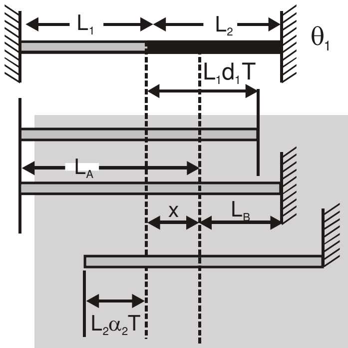
t = duration time
1 day = 24 × 3600 sec.

$$\therefore \quad \Delta t = \frac{1}{2} \times \frac{0.000036}{3} \times 20 \times 24 \times 3600$$

$\Delta t = 10.368$ sec.

**Note :** If we increase temperature then time period increases and watch becomes slow.

**8.** $L_A = L_1 + x$




$L_B = L_2 - x$

$$Y = \frac{F \times \ell}{Ae}$$

$$e = \frac{F \times \ell}{YA} = \text{extension or compression due to force}$$

$$e = L_1\alpha_1 T - x = \frac{FL_1}{Y_1 A} \quad \text{..............(1)}$$

$$e = L_2\alpha_2 T + x = \frac{FL_2}{Y_2 A} \quad \text{..............(2)}$$

By adding equation (1) and equation (2)

We get $F = \dfrac{AT(L_1\alpha_1 + L_2\alpha_2)Y_1Y_2}{L_1Y_2 + L_2Y_1}$ **Ans.**

By dividing (1) and (2)

We get $x = \dfrac{L_1L_2T(Y_1\alpha_1 - Y_2\alpha_2)}{L_1Y_2 + L_2Y_1}$

So, $L_A = L_1 + x$; $L_B = L_2 - x$ **Ans.**

**9.** $\Delta\ell = 1.91 \text{ mm} = \Delta\ell_1 + \Delta\ell_2$

$\Rightarrow$ $0.191 \text{ cm} = \ell_1\alpha_1\,\Delta\theta + \ell_1\alpha_2\,\Delta\theta.$

$\Rightarrow$ $0.191 = (30 \times 17 \times 10^{-6} + 70\,\alpha_2) \times 100$

$\alpha_2 = 2 \times 10^{-5}$ **Ans.**

$Y = \dfrac{F}{A\alpha\Delta\theta}$

$F_1 = F_2$

$Y_1 A\,\alpha_1\Delta\theta = Y_2 A\,\alpha_2\,\Delta\theta.$

$\therefore\ Y_2 = \dfrac{Y_1\alpha_1}{\alpha_2} = \dfrac{1.3\times10^{11}\times1.7\times10^{-5}}{2\times10^{-5}} = 1.105 \times 10^{11}\ \text{N/m}^2$ **Ans.**

**10.** $W_0 = mg = 46$ g wt

at $\theta_1 = 27^\circ$ C

$W_1 = 30$ gm wt $= W_0 - B_1$ $\Rightarrow$ $B_1 = (46 - 30)$gm $\Rightarrow$ $B_1 = 16$gm wt $= V_1\,\rho_1 g$

at $\theta_2 = 42^\circ$ C

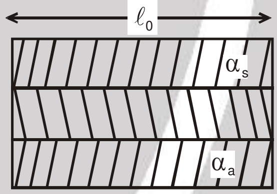
$W_2 = 30.5$ g wt $= W_0 - B_2$ $\Rightarrow$ $B_2 = 15.5$ gm wt $= V_2\,\rho_2 g$

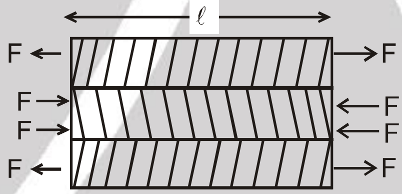
$\therefore\ \dfrac{B_2}{B_1} = \dfrac{V_2\rho_2}{V_1\rho_1}$

$\dfrac{15.5}{16} = (1 + 3\alpha_s \times 15) \times \dfrac{1.2}{1.24}$

$\alpha_s = \left[\left(\dfrac{15.5}{16}\times\dfrac{1.24}{1.2}\right)-1\right]\times\dfrac{1}{45}$ $\alpha_s = 2.31 \times 10^{-5}$ /ºC $= \dfrac{1}{43200}$ / **ºC** **Ans.**

**11.** at 0 ºC




at θ ºC

If rods are free to expand.

$\ell_s = \ell_0\,(1 + \alpha_s\theta)$

$\ell_a = \ell_0\,(1 + \alpha_a\theta)$

$Y = \dfrac{F/A}{x/\ell}$

$x = \dfrac{F\times\ell}{AY}$ if $\alpha_s > \alpha_a$

for steel(compression) $x = \dfrac{2F\times\ell_0}{AY_s} = \ell_0\,(1 + \alpha_s\Delta\theta) - \ell$ .....(i)

for aluminium (expansion) $x = \dfrac{F\times\ell_0}{AY_a} = \ell - \ell_0\,(1 + \alpha_a\Delta\theta)$ .....(ii)

by solving (i) and (ii)
we get

then $\ell = \ell_0\left[1+\dfrac{2Y_a\alpha_a + Y_s\alpha_s}{2Y_a + Y_s}\ \theta\right]$ **Ans.**

**12.** (a) $\ell = 1\{1 + 2 \times 10^{-5} \times 20\}$

(b) $\ell = 10\{1 - 4 \times 10^{-4}\}$

(c) $\ell = 10\{1 + 4 \times 10^{-4}\}$

(d) $\%\ \ell_1 = -4 \times 10^{-2}\ \%$

$\%\ \ell_2 = \dfrac{-4 \times 10^{-2}}{1 + 4 \times 10^{-4}}\ \% \approx -4 \times 10^{-2}\ \%$

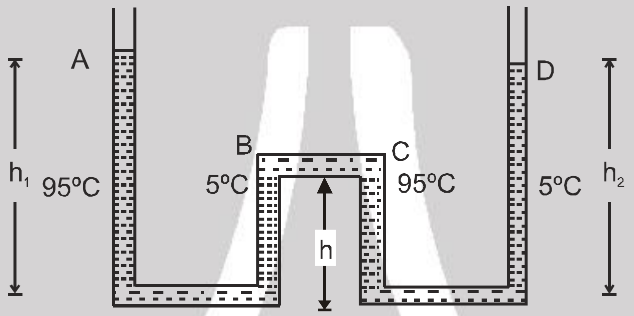
(e) $\ell_1 = 10\{1 + 20 \times 4 \times 10^{-5}\}$
$\ell_2 = 10\{1 - 20 \times 4 \times 10^{-5}\}$

(f) $\ell_1 = 10\{1 + 40 \times 10^{-5}\}$
$\ell_2 = 10\{1 - 40 \times 10^{-5}\}$

**13.** Density of a liquid varies with temperature as–

$$\rho_{t^\circ C} = \left(\frac{\rho_{0^\circ C}}{1 + \gamma\ \ t}\right)$$

Here $\gamma$ is the coefficient of volume expansion of liquid.




In the figure –
$h_1$ = 52.8 cm, $h_2$ = 51 cm and h = 49 cm
Now pressure at B = pressure at C.

Therefore

$P_0 + h_1 \rho_{95^\circ} g - h \rho_{5^\circ} g = P_0 + h_2 \rho_{5^\circ} g - h \rho_{95^\circ} g$

$\Rightarrow \quad \rho_{95^\circ}(h_1 + h) = \rho_{5^\circ}(h_2 + h)$

$\Rightarrow \quad \dfrac{\rho_{95^\circ}}{\rho_{5^\circ}} = \dfrac{h_2 + h}{h_1 + h}$

$\Rightarrow \quad \dfrac{\dfrac{\rho_{0^\circ}}{(1 + 95\gamma)}}{\dfrac{\rho_{0^\circ}}{(1 + 5\gamma)}} = \dfrac{h_2 + h}{h_1 + h}$

$\Rightarrow \quad \dfrac{1 + 5\gamma}{1 + 95\gamma} = \dfrac{51 + 49}{52.8 + 49} = \dfrac{100}{101.8}$

Solving this equation, we get

$\gamma = 2 \times 10^{-4}$ per ºC